# Nonlinear Lamb wave mixing in prestressed plates

**Meng Wang***, Annamaria Pau**

*Dept. Astronautical, Electrical and Energy Eng., Sapienza Univ. of Rome, Rome, Italy, ORCID # 0000-0003-2946-0185

**Dept. Structural and Geotechnical Eng., Sapienza Univ. of Rome, Rome, Italy, ORCID # 0000-0002-4946-0302

**Abstract**. We address the nonlinear wave propagation in a homogeneous and isotropic prestressed elastic plate, accounting for material and geometric nonlinearities. Two internally-resonant Lamb wave modes of different frequencies ($\omega_1$ and $\omega_2$) are allowed to mix within the material, which generates a third type of harmonic waves at frequencies ($\omega_1 \pm \omega_2$). In this study, we investigate the time and space evolution of the resonant higher armonics for the one-way resonant mixing of waves using a finite element model. We employ also multiple-scale methods to verify some phenomena emerging from the numerical analysis. The influence of prestress on the wave mixing resonance conditions is elucidated by observing the material nonlinear parameter, which proved to be quite effective in the detection of a change in the state of stress.

## Introduction

Among the various nonlinear ultrasonic NDE techniques, the method of second harmonic generation (SHG) takes advantage of the spatially cumulative nature of the second harmonic generated by a propagating wave. Various studies have been conducted using the method of SHG to quantitatively evaluate fatigue and creep damage in the early stages [1, 2]. When two waves propagate in a nonlinear medium, their interaction generates a third wave, called mixed wave. If these two primary waves satisfy certain resonance conditions, the mixed wave is also a propagating wave, and its maximum amplitude is proportional to the size of the mixing zone and to the distance travelled. We have in this case a wave triplet. It happens that two identical propagating longitudinal waves and their resonant second harmonic are such a triplet [3]. The generation of the second harmonic is simply the resonant mixing of two identical propagating longitudinal waves, or self-mixing. In the technique called one-way mixing, two primary waves propagating in the same direction are employed. Here, we conduct numerical finite-element simulations to investigate the spatial variation of higher harmonics and mixed waves generated in the one-way mixing, and elucidate the dependence of these amplitudes on the initial prestress. Multiple-scales analytical techqniques are also used to motivate some unexpected numerical results.

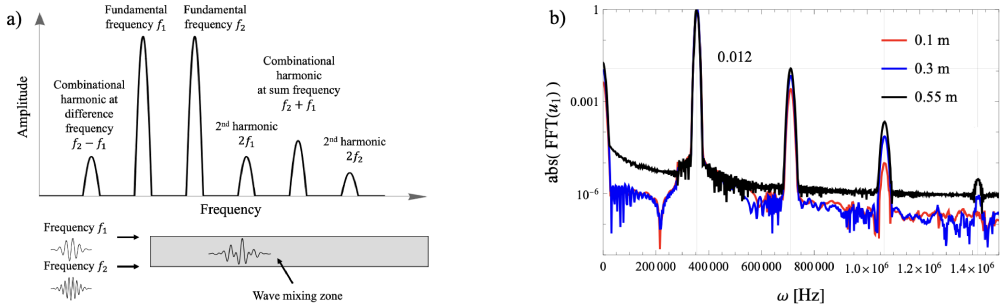


Figure 1: (a) Schematic diagram of frequency spectrum for ultrasonic guided wave mixing, and wave mixing zone in plate. (b) Normalized spectrum of S1-S2 Lamb wave modes at different distances from the excitation boundary in resonance condition.[4]

## Results and discussion

The problem considered concerns the propagation of waves in a prestressed Aluminum plate. The finite-element model employed is a second-order plane-strain approximation accounting for material and geometric nonlinearities, considering different states of prestress. It is shown that, for internally-resonant S1 and S2 modes, the nonlinear parameter increases with propagation distance and when the initial stress increases. Besides the well-known internal resonance S1-S2, other resonances due to reciprocal interaction and mixing of these two waves at integer multiples of the primary frequency can be observed. In fact, multiple higher resonance conditions exist, though these higher resonances were not reported in the literature, because their occurrence cannot be predicted by the classical two-scales perturbation analysis. Here, their existence is demonstrated and their characteristics are outlined resorting to the method of multiple scales. The one-way mixing involving different combinations of resonant frequencies is investigated, in view of their potential application to the sensing of the state of stress.

## References

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