A parsimonious identification approach in the frequency-domain for experimental fractional systems of unknown order

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Abstract. When the order of experimental fractional systems is unknown, frequency domain identification is typically based on linear least-squares (L_2) methods, which often prove unsatisfactory. The main reason is that the solutions they provide are dense, often leading to a continuous of identified fractional differential orders, which obfuscate the very basic phenomena one wishes to highlight. In order to overcome such difficulty, the present work proposes the use of a L_1 based nonlinear identification approach, which naturally leads to parsimonious identified results. This method is successfully illustrated on a system transfer function with both integer and fractional differential terms.

Introduction

Fractional models are being increasingly used in many branches of physics and engineering [1]. In this work we deal with the identification of fractional models from experimental Frequency Response Functions (FRFs) in the frequency domain $H(i\omega)$, when the integro-differential equations discrete order(s) α_n are unknown:

$$Z(i\omega) = \frac{1}{H(i\omega)} = \frac{F(i\omega)}{X(i\omega)} = \sum_{n=1}^{N} c_n (i\omega)^{\alpha_n}$$

The linear identification procedure proposed by Hartley & Lorenzo [2] is straightforward to implement. The differential order continuum is discretized in the interval $[\alpha_{\min}, \alpha_{\max}]$ sampled at $\Delta\alpha$, so that $m = 1, \cdots, M$ differential order terms are identified. The FRFs $Z(i\omega_k) = 1/H(i\omega_k)$ are measured at $k = 1, \cdots, K$ frequencies. Then, the model is formulated in and solved by the Least Squares Deviation (LSD) method:

$$\begin{bmatrix} (i\omega_1)^{\alpha_1} & \cdots & (i\omega_1)^{\alpha_M} \\ \vdots & \ddots & \vdots \\ (i\omega_K)^{\alpha_1} & \cdots & (i\omega_K)^{\alpha_M} \end{bmatrix} \begin{Bmatrix} c_1 \\ \vdots \\ c_M \end{Bmatrix} = \begin{Bmatrix} Z(i\omega_1) \\ \vdots \\ Z(i\omega_K) \end{Bmatrix} \rightarrow \mathbf{Mc} = \mathbf{z} \rightarrow \mathbf{c}_{LSD} = \min \left\| \mathbf{Mc} - \mathbf{z} \right\|_{L2} \rightarrow \mathbf{c}_{LSD} = \mathbf{M}^{+}\mathbf{z} = (\mathbf{M}^T\mathbf{M})^{-1}\mathbf{M}^T\mathbf{z}$$

Results and discussion

Unfortunately, solutions provided by the LSD method are dense, producing a continuous of identified fractional differential orders as M increases, which is objectionable. In order to overcome this difficulty, we propose to replace the LSD by the Least Absolute Deviation (LAD) method, a nonlinear L1-based identification approach that is sparsity-promoting, leading to parsimonious results:

$$\mathbf{c}_{LAD} = \min \left\| \mathbf{Mc} - \mathbf{z} \right\|_{L1}$$

We illustrate the identification results on the following test system, previously used by Hartley & Lorenzo [2]:

$$Z(\omega) = 1(i\omega)^2 + 1.4(i\omega)^{1.5} + 1(i\omega) + 1.4(i\omega)^{0.5} + 1$$

with three integer and two fractional derivatives. The range of integro-differential order hypothesized for identification is $\alpha \in [-1\,,\,3]$, sampled at $\Delta\alpha = 0.1$, these conditions being highly stringent compared to [2]. The LAD identification is based on the Iterative Reweighted Least Squares (IRLS) algorithm [3], which can minimize any Lp norm. Figure 1 clearly demonstrates, not only the essential problem of the common LSD identification method, but also the significant improvements obtained when the sparsity-promoting LAD approach is used. Robustness of the identification results is currently being addressed.

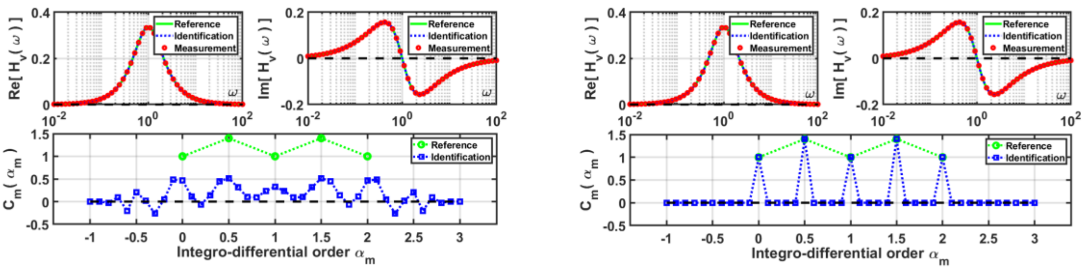

Figure 1: Left side - FRFs and derivative coefficients using common least-squares (L_2) identification; Right side - FRFs and derivative coefficients using nonlinear sparse (L_1) identification.

References

[1] Machado J.A. et al. (2010) Some Applications of Fractional Calculus in Engineering. *Math. Probl. Engineering* **2010**/639801:1-34.

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[3] Björk A. (1996) Numerical Methods for Least Squares Problems. SIAM, Philadelphia.