

# Parametric resonance caused by mass imbalance on railway wheels

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**Abstract.** This study investigates the effect of mass imbalance of the wheelset. By employing a 2-DOF mathematical model, it is theoretically shown that the effect produces parametric resonance and decreases the critical speed of the hunting motion. Also, the theoretically predicted phenomena are experimentally observed by using a simple apparatus.

## Introduction

In recent years, the dynamics of railway vehicles has been actively studied to improve their performance for safety and comfort. To this end, it is essential to clarify the factors behind various phenomena occurring in railway vehicles. For example, when a rail vehicle wheelset exceeds a certain running speed, it swaies from side to side. This resonance is well known as hunting motion that is a self-excited oscillation by non-conservative contact forces acting between the wheel and the rail [1]. The effects of wheel and brake disc mass imbalance and rail surface roughness on the dynamics have also investigated. Szabo and Lorant [2] analyzed the linearized equations of motion of a railway vehicle by taking into account a time-varying coefficient excitation term expressing the imbalance in the governing equations. It is numerically shown that the critical speed for hunting motion decreases with increasing the imbalance. However, the characteristic of the instability phenomenon has not analytically or experimentally investigated. In this presentation, we clarify that theoretically and experimentally. Figure 1 shows a single railway wheelset with 2-DOF model considering the lateral and yaw motions  $y$  and  $\psi$ . The equations of motion for the lateral and yaw motions are expressed as [3]

$$M \frac{d^2 y}{dt^2} + \frac{2\kappa_{yy}}{v} \frac{dy}{dt} + (2k_y + \frac{2Q\gamma}{d_0})y - 2\kappa_{yy}\psi = 0, \quad (1)$$

$$I \frac{d^2 \psi}{dt^2} + \frac{2d_0^2 \kappa_{xx}}{v} \frac{d\psi}{dt} + \frac{2d_0 \kappa_{xx} \gamma}{r_0} y + 2k_x d_1^2 \psi = 0, \quad (2)$$

where  $\gamma$  is tread angle which is assumed large,  $t$  is the time,  $v$  is the running speed,  $M$  is the wheelset mass,  $I$  is the wheelset momoent of inertia around the  $z$ -axis,  $d_0$  is the half-track gauge,  $d_1$  is the distance between support springs,  $r_0$  is the centered wheel rolling radius,  $k_x$ ,  $k_y$ ,  $\kappa_{xx}$ ,  $\kappa_{yy}$  are the longitudinal and lateral stiffness and creep coefficients, respectively, and  $Q$  is the wheel load.

## Results and discussion

In order to understand the effects of imbalance and the qualitative non-linear characteristics on the post destabilized states, we consider the periodic changes in wheel load and creep coefficients and introduce the third order non-linear restoring force in the lateral direction as a representative nonlinear force into Eqs. (1) and (2). Then, we theoretically analyze the nonlinear equations of motion by using a reduction method and the method of multiple scales. As a result, we obtain the variation of the critical speed depending on the magnitude of imbalance and the steady state amplitude in the state where the speed is above the critical one. Furthermore, the theoretically predicted phenomena were experimentally confirmed by using a roller rig and a wheelset device.

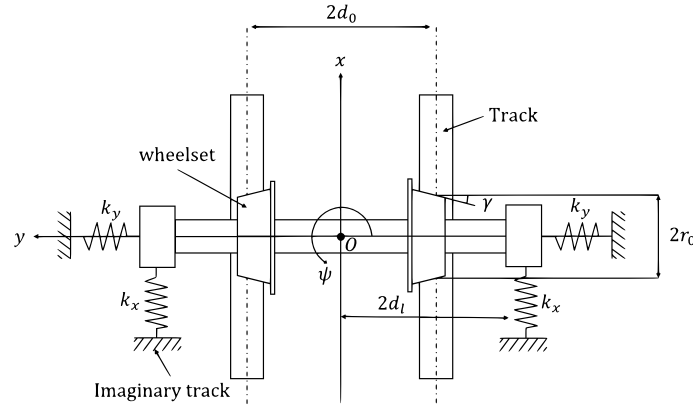


Figure 1: Two degree of freedom model of railway wheelset.

## References

- [1] S. Iwnicki (2003) Simulation of wheel-rail contact forces, *Fatigue Fract. Eng. Mater. Struct.* **26**:887-900.
- [2] Z. Szabo, G. Lorant(2000) Parametric excitation of a single railway wheelset, *Veh. Syst. Dyn.* **33**:49-55.
- [3] K. Popp (1997) Parametric excitation of a wheelset, *ZAMM Z. angew. Math. Mech.* **77**:269-270.