Delay-embedded modal analysis for spectral submanifold identification

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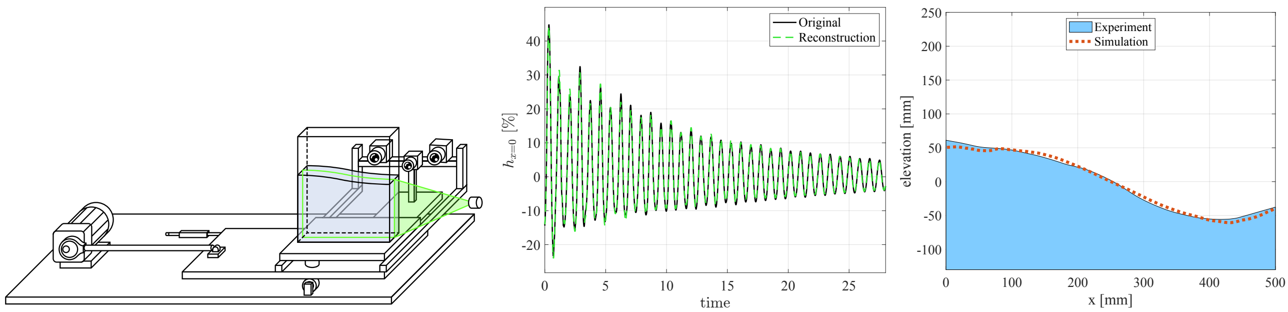
Abstract. We show that in a delay-embedded space, the linearized dynamics at a fixed point can be computed solely from the eigenvalues of the full linearized system independent of its eigenvectors. This observation provides guidelines for choosing the delay embedding parameters. It also implies that the tangent space of a delay-embedded spectral submanifold (SSM) is fully determined by the spectrum of the corresponding eigenspace. Thus, we can facilitate the identification of SSMs from data by prescribing their tangent spaces based on eigenvalue estimates. Applying this procedure to data from tank sloshing experiments, we identify a 6D SSM and correctly predict the system's multimodal decay.

Introduction

In data-driven model identification, delay embedding is routinely used. Examples of model reduction methods based on delay embedding include eDMD [1], HAVOK [2], and false nearest neighbors [3]. Another approach where delay embedding has been employed is data-driven model reduction to SSMs [4, 5]. Takens' embedding theorem states that delay-embedding a signal from a generic observable function on the full phase space at least $2d + 1$ times recovers d-dimensional invariant objects. In practice however, their identification crucially depends on choosing the timelag and embedding dimension properly. Improved understanding of how invariant objects are reconstructed in observable spaces can thus aid model order reduction methods significantly.

Results and discussion

We show that on a delay-embedded SSM, the linear part of the dynamics is fully determined by the eigenvalues of its spectral subspace. Therefore, when the eigenvalues of interest are known, we can facilitate computation of a delay-embedded SSM by prescribing the tangent space, even if the observable function and the SSM in the full phase space are unknown. We apply this method to data from sloshing experiments, where a tank partially filled with water is mounted on a moving platform horizontally excited by a motor, and the surface profile of the fluid is recorded (Figure 1a) [4]. As the forcing amplitude increases, more sloshing modes are activated.



(a) (b) (c)

Figure 1: (a) Experimental setup for tank sloshing. (b) Measurement and 6D SSM prediction of the leftmost point on the surface profile. (c) SSM prediction of full surface profile.

While previous work successfully captured the dynamics of the first mode with a 2D SSM [4, 5], here, we model the mutlimodal decay on a 6D SSM. The key technology allowing this enhancement is the enforcement of the tangent space in our SSM reconstruction, based on the theoretically known first three eigenfrequencies. Figure 1b shows good agreement between the experimental surface profile elevation at the tank wall and the delay-embedded SSM-reduced prediction. Furthermore, our 6D reduced model accurately predicts the full surface profile decay in Figure 1c. Finally, our theory on delay-embedded modal analysis provides insight into optimal parameter choice applicable to any model reduction method.

References

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