Observer design for semi-linear stochastic partial differential equations

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Abstract. In this work an observer design for semilinear stochastic partial differential equations (SPDEs) involving Lèvy type noise is presented. By constructing an appropriate Lyapunov functional, a new set of sufficient conditions are obtained to guarantee the mean square exponential stability (MSES) of the error system.

Introduction

Observers are ingenious tool to estimates the system states which helps to reconstruct the system states for more appropriate performance. In practice random noises are unavoidable, while dealing with such cases SPDEs are most helpful model. Particularly, Lèvy process is more interesting because in this, noise splittable as continuous and discontinuous parts. Few results are found in literature on MSES, boundary control for reaction diffusion systems with Brownian motion [1, 2, 3]. With our best knowledge observer based stability analysis of SPDEs not yet investigated. Motivated by above discussions, the main objective of the present work is to design an observer-based boundary control for the considered semilinear SPDE and to derive the sufficient conditions for the MSES of the error system in terms of LMIs with the help of Lyapunov theory.

System description

Consider the following semilinear stochastic parabolic type equation:

$$dy(x,t) =\{A\frac{\partial^2 y}{\partial x^2} + By(x,t) + f(t,y(x,t))\}dt + \sigma(t,y(x,t))dW(t) + \int_Z \phi(t,y(x,t),z)\tilde{N}(dt,dz),$$

where $y(x,t)$ is the state vector with space variable x and time variable t. Boundary conditions are $\frac{\partial y}{\partial x}\Big|_{x=0} = 0$, $\frac{\partial y}{\partial x}\Big|_{x=1} = u(t)$, output of the system $\bar{y}(x,t) = Cy(x,t)$. $u(t)$ is boundary control input to be design. 'A' is known constant diagonal matrix and B, C are known constant matrices. $W(t)$ is a one dimensional standard Brownian motion and $\tilde{N}(dt,dz) = N(dt,dz) - \lambda(dz)dt$ is compensated Poisson measure with intensity measure $\lambda(dz)$. $f(\cdot)$, $\sigma(\cdot)$, $\phi(\cdot)$ are semilinear functions satisfies Lipschitz condition. State observer system is considered as

$$d\hat{y}(x,t) = \{A\frac{\partial^2 \hat{y}}{\partial x^2} + B\hat{y}(\cdot) + \hat{f}(\cdot) + L(\bar{y}(\cdot) - \hat{\bar{y}}(\cdot))\}dt + \hat{\sigma}(\cdot)dW + \int_Z \hat{\phi}(\cdot)\tilde{N}(dt,dz),$$

where $\hat{y}(x,t)$ is estimated state and L is observer gain. Boundary control $u(t) = K\hat{y}(1,t)$, where K is control gain. Let error state be $e(x,t) = y(x,t) - \hat{y}(x,t)$. By constructing Lyapunov functional $V(\cdot) = \int_0^1(y^T(x,t)P_1y(x,t) + e^T(x,t)P_2e(x,t))dx$, main result is stated below.

Theorem: For given scalar $\alpha > 0$, $c_1, c_2, q_1, q_2, F_1, F_2$, there exist symmetric positive definite diagonal matrices P_1, P_2, scalars $\bar{\rho}_1$, $\bar{\rho}_2$, ϵ_1, ϵ_2 and appreciate matrices $\mathcal{K}$, $\mathcal{L}$ the LMIs $[\Xi]_{6\times 6} < 0,\ P_1 \le \bar{\rho}_1 I,\ P_2 \le \bar{\rho}_2 I$ hold with $\Xi = 2P_1B + \bar{\rho}_1c_1I + \bar{\rho}_1q_1I - \frac{2\pi^2}{2}P_1A + 2\alpha P_1 - \epsilon F_1^TF_2$, $\Xi_{12} = \frac{\pi^2}{4}(P_1A)^T$, $\Xi_{13} = P_1^T + \frac{1}{2}\epsilon(F_1+F_2)I$, $\Xi_{22} = 2A\mathcal{K} - \frac{2\pi^2}{4}P_1A$, $\Xi_{25} = (A\mathcal{K})^T$, $\Xi_{33} = \Xi_{66} = -\epsilon I$, $\Xi_{44} = 2P_2B + \bar{\rho}_2c_2I + \bar{\rho}_2q_2I - \frac{2\pi^2}{2}P_2A + 2\mathcal{L}C + 2\alpha P_2 - \epsilon F_1F_2$, $\Xi_{46} = P_2^T + \frac{1}{2}\epsilon(F_1+F_2)I$, $\Xi_{45} = \frac{\pi^2}{4}(P_2A)^T$, $\Xi_{55} = -\frac{2\pi^2}{4}P_2A$ and other terms are zero. Then, error system is MSES and control & observer gains are $K = P_1^{-1}\mathcal{K}$, $L = P_2^{-1}\mathcal{L}$, respectively.

Numerical example

Letting $f(t,y(x,t)) = y(x,t)(B - y(x,t))$, the considered system represents Fisher's equation. By fixing $A = diag\{0.4, 0.7\}$, $B = \begin{bmatrix} -0.51 & 0.065 \\ 0.2 & -0.9 \end{bmatrix}$, $C = \begin{bmatrix} -0.6 & -0.3 \\ 0.01 & -0.2 \end{bmatrix}$, we get the gains $K = \begin{bmatrix} 0.7802 & 0 \\ 0 & 0.5606 \end{bmatrix}$, $L = \begin{bmatrix} 0.2286 & -37.7459 \\ 18.9340 & -26.2291 \end{bmatrix}$, by solving the LMIs in proposed theorem. The sector nonlinearity has bounds $[F_1, F_2] = [-0.002, 0.002]$.

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