Reliability Problem of a Fractional Stochastic Dynamical System Based on Stochastic Averaging Method and Deep Learning Algorithm

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Abstract. this paper consider a generalized Van der Pol system with fractional derivatives excited by a white Gaussian noise. Firstly, a generalized harmonic transformation is used to get an approximated expression. Applying stochastic averaging methods with energy envelopes to obtain the Ito differential equations and obtaining the Kolmogorov backward equations (KBE) related to the system energy. Then, combining Monte Carlo sampling to perform data-driven and neural network, a new algorithm is obtained to solve the reliability function that satisfies KBE, which is the innovation of this paper. The algorithm does not need boundary conditions and reduces the need for data volume in high-dimensional problems.In addition, the algorithm is applied to obtain the stationary probability density function of the system.

Introduction

Engineering structure with viscoelastic material are generally modelled by a fractional-order system[1] , the reliability problem of relative structural vibration under random excitations is always a hot issue in the field of the stochastic dynamical systems. Since the KBE equation satisfied by the reliability function cannot be solved exactly, in the previous work, methods such as stochastic averaging[2] and Monte Carlo[3] are often used to obtain numerical simulation solutions.Based on the fact that deep learning algorithms can show strong accuracy in fitting functions, this paper provides a new algorithm for solving reliability functions.

Results and discussion

this paper uses the deep learning algorithm based on neural network to study the reliability problem of the fractional stochastic dynamical system.

$$\ddot{x} + \left(\beta_1 - \beta_2 x^2 + \beta_3 x^4\right)\dot{x} + \omega_0^2 x + \varepsilon D^{\alpha} x\left(t\right) = W\left(t\right) \tag{1}$$

The innovation of our deep learning is to use the training sample coming from the combination of Monte Carlo simulation to the system with some certain reference points, which has the advantage of grid-less structure and ability to solve high-dimensional systems. Its loss function is composed by two parts and shown below:

$$L\left(\theta\right) = \frac{1}{N_1}\sum_{i=1}^{N_1}\left(\mathcal{L}\tilde{u}\left(x_i,\theta\right)\right)^2 + \frac{1}{N_2}\sum_{j=1}^{N_2}\left(\tilde{u}\left(y_j,\theta\right) - v\left(y_j\right)\right)^2 \tag{2}$$

In which the first part is the error caused by the neural network solution fitting the Backward Kolmogorov equation at the training point, the second part is the inaccuracy between the neural network solution at the reference points and the Monte Carlo approximate solutions. It is concluded that the reliability probability is monotonously decreasing with the time increasing, and the results derived from the stochastic averaging method and deep learning algorithm are nearly agreement with the accordant tendency.

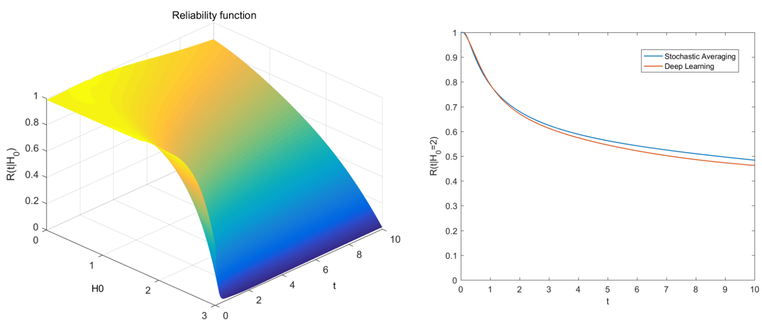

Figure 1: Reliability function derived from stochastic averaging method and deep learning algorithm

References

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