## Nonlinear vibrations of a composite circular plate with a concentrated mass: effects of equilibrium configurations

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Abstract. Nonlinear vibrations of a composite circular plate with a concentrated mass are investigated with the focus on the effects of curved equilibrium configurations caused by weights. A mathematical model is derived from the generalized Hamiltonian principle considering the weights. The Galerkin method is applied to determine equilibrium configurations and the results are validated by the finite element method. The fundamental frequency and the steady-state responses are respectively calculated in free vibrations and harmonically forced vibrations. The results demonstrate that the weights have more significant effects on the curved equilibrium configurations for the larger substrate radius, the larger concentrated mass, and the smaller upper and lower radii. The account for the weights increases the fundamental natural frequency, and the relative difference increases with the increasing weights.

## Introduction

Circular plates with concentrated masses have been used in different industries. They are often exposed to severe dynamic loading resulting to large vibrations. Thus, it is significant to investigate nonlinear vibrations of circular plates with concentrated masses. Currently, nonlinear vibrations of circular plates with masses have been studied by Huang [1], Li et al. [2], Yuan et al. [3], and other scientists. When a circular plate with concentrated masses vibrates nonlinearly, the weight affects the equilibrium configuration, and the changing configuration in turn affects vibration frequencies and shapes. Several investigations have demonstrated the crucial role of weights. Chen et al. [4] investigated the effects of the weight of the nonlinear energy sink (NES) on structural vibrations. They found that the weights should be accounted for when the primary structure is excited by small excitations or coupled with large NES masses. However, the effects of the weights on nonlinear vibrations of circular plates with masses are rare in the literature. This is the motivation here.

## **Results and discussion**

The generalized Hamiltonian principle is applied to obtain the governing equation with boundary conditions. Equilibrium configurations and a discretization model for vibrations around the configurations are derived from the Galerkin method. The model can be used to determine the fundamental frequency and the steady-state responses. The investigation yields the following findings: (1) The account of the weights introduces additional linear and quadratic terms to the governing equation and weakens the hardened nonlinearity. (2) The static equilibrium configuration increases with the increasing concentrated mass, substrate radius and the decreasing upper and lower radii. (3) In free vibrations, the curved equilibrium configuration caused by the weights increases the fundamental natural frequency, and the relative difference increases with the increasing curved equilibrium configuration. (4) In harmonically forced vibrations, the curved equilibrium configuration increases the resonant amplitude, and the absolute difference increases with the increase of the curved equilibrium configuration.



Figure 1: Natural frequencies and resonant amplitudes for varying parameters: (a-b) variation of natural frequencies and the relative difference with the concentrated mass, the substrate radius, and the upper and lower radii; (c-d) variation of resonant amplitudes and the absolute difference with the concentrated mass, the substrate radius, and the upper and lower radii.

## References

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