Identification of Secondary Resonances using a Control-based Method

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Abstract. Recently, advanced experimental measurement techniques such as phase-locked-loop feedback control and control-based continuation have been developed, mainly for identifying primary resonances. The objective of the present study is to characterize secondary resonances by taking advantage of adaptive digital filters, a powerful tool that can be incorporated as a building block in the control loop. Adaptive filters allow the experimenter to perform online Fourier decomposition so that the Fourier coefficients of the harmonic components of interest can be estimated at each time instant. The phase properties of secondary resonances are then exploited for the identification of the associated frequency response and backbone curves. It is demonstrated that the phase resonances of odd and even superharmonic resonances of a nonlinear structure can be effectively targeted. The designed testing scheme is found to stabilize the unstable orbits and circumvent the problems induced by bifurcations.

Introduction

Control-based vibration testing methods have shown promise in identifying folded and unstable responses of nonlinear systems. These methods can provide more insights into the dynamics as compared to the conventional testing approaches (i.e., without the use of a controller). Most existing studies focus on the characterization of primary resonances of mechanical systems, which can feature different types of nonlinearity [1, 2]. However, multi-harmonic responses can be activated with a harmonic excitation, which, in turn, can trigger the excitation of secondary resonances such as superharmonic and subharmonic resonances.

The objective of this study is to develop a control-based testing scheme which can characterize secondary resonances. To this end, we resort to phase-locked loops (PLLs) coupled to adaptive digital filters, which allow the experimenter to perform online Fourier decomposition. The periodic response is fitted with a truncated Fourier series $x(t) = \sum_{n=1}^{N} \hat{x}_n \sin(n\Omega t + \phi_n)$, where ϕ_n is the phase lag of n-th harmonic. PLL feedback control is first implemented to identify the frequency response curves of the secondary resonance of interest, by exploiting the monotonous evolution of the phase lag between the harmonic of interest and the forcing. PLL can also track backbone curves based on the phase resonance criterion in [3]. The frequency of the harmonic excitation $f(t) = \hat{f} \sin(\int_0^t \Omega(\tau)\mathrm{d}\tau)$ acting on the structure is determined by a PI controller, with $\Omega(t) = \Omega_0 + K_P(\Phi_{\mathrm{ref}} - \Phi_n(t)) + K_I \int_0^t (\Phi_{\mathrm{ref}} - \Phi_n(\tau))\mathrm{d}\tau$. Here, Φ_{ref} is the assigned reference phase.

Results and discussion

The algorithm is first tested on a Duffing oscillator, $\ddot{x} + 0.001\dot{x} + x + x^3 = f$, in a virtual experiment. As shown in Figure 1, there is a good agreement between the results computed by the harmonic balance method and PLL testing. The responses exhibiting bifurcations can be effectively tackled by adjusting Φ_{ref}, thanks to the use of the adaptive filter and feedback control. The backbone curves of the 3:1 and 2:1 resonances are obtained by setting Φ_{ref} to be $-\pi/2$ and $-3\pi/4$, respectively. The phase resonance criterion can predict both resonance frequencies and amplitudes accurately.

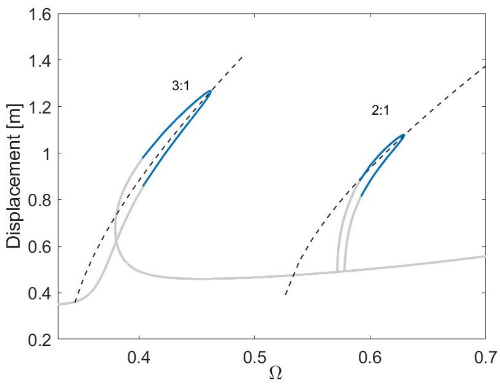

Figure 1: Frequency response curves (blue) and backbone curves (black) of 3:1 and 2:1 resonances identified by PLL for $\hat{f} = 0.4$ N. The reference solution is provided by the harmonic balance method (grey).

References

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