Fast computation and characterization of forced response surface of high-dimensional mechanical systems via spectral submanifolds and parameter continuation

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Abstract. Forced response curves (FRCs) have been widely used to characterize the nonlinear dynamics of mechanical systems subject to periodic excitations. Forced response surfaces (FRSs), which depict the nonlinear forced response over a range of excitation amplitudes, however, have been rarely computed in the literature. FRSs remove the need for a case-by-case computation of FRCs over a sample of excitation amplitudes and automatically uncover any isolas in the forced response, that are otherwise hard to predict. Here, we construct spectral submanifold-based reduced-order models (ROMs) of high-dimensional mechanical systems and equip these ROMs with multidimensional manifold continuation of fixed points to efficiently extract FRSs. By solving optimization problems on these ROMs, we also show how to extract the ridges and valleys in an FRS, which delineate the main physical features of the forced response. We demonstrate fast and effective FRS computation using the proposed approach over finite-element models of structural systems.

Introduction

We consider a periodically forced nonlinear mechanical system with forcing frequency Ω and forcing amplitude ϵ. Let $\mathcal{A}$ be the amplitude of the periodic orbit. The frequency response surface (FRS) is a two-dimensional manifold in the space $(\Omega, \epsilon, \mathcal{A})$ that is foliated by the FRCs. Ridges and valleys in the surface present the skeleton of the response surface. In addition, the projection of them onto the plane $(\Omega, \mathcal{A})$ gives the *damped backbone curve*. The ridges and valleys are obtained as a collection of local extrema of the one-parameter family of FRCs under variation in ϵ. Since covering a two-dimensional manifold is much more demanding than that of a one-dimensional manifold, one can use ridges and valleys to characterize the main features of the FRS without computing it. However, locating these ridges and valleys are still computationally challenging for high-dimensional problems.

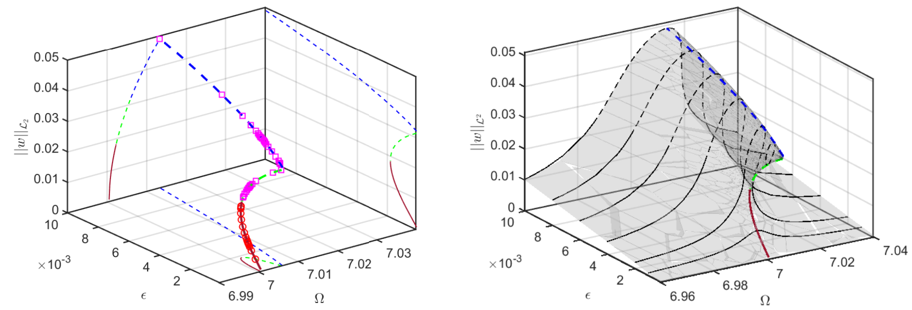

Figure 1: Left: Ridges and valleys obtained from SSM-based ROM(lines) and the full system via collocation (markers). Right: FRS and its the ridges and valleys obtained via the SSM-based ROM along with sampled FRCs obtained from the full system via collocation.

Results and discussion

We remove this bottleneck using reduced-order models (ROMs) on spectral submanifolds (SSMs) [1]. Specifically, we reformulate the periodic orbits of the full system as fixed points of their low-dimensional ROMs on SSMs computed via SSMTool [2]. We then use multidimensional manifold continuation [3] of these fixed points to compute the FRS of the full system. Furthermore, we use a successive continuation [4] to locate the ridges and valleys directly with the computation of only one forced response curve.

We apply the proposed method to a cantilever beam with a nonlinear support. This system is discretized with 50 DOFs. As seen in Fig. 1, the results from SSM predictions match well with that of collocation methods. Here, the complete FRS is obtained in less than half an hour via the SSM reduction, while the 6 sampled FRCs are obtained in nearly 6 hours using a collocation method on the full system. Furthermore, generating the ridges and valleys via the SSM reduction took 31 seconds whereas that using a collocation scheme on the full system took 1.5 days. Note that the isolas are uncovered automatically via this FRS computation, as shown in Fig. 1.

References

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