On the Dynamics Analysis of Microresonator System with Fractional-order

Tao Xi *, **Jin Xie** * and Zhaohui Liu*

School of Mechanical Engineering, Southwest Jiaotong University, Chengdu, Sichuan, P.R. China

Abstract. It is observed that the fractional derivative model is more accurate in describing the properties of viscoelastic materials, a fractional differential dynamics model of a microresonator is established using Rayleigh-Ritz method, Lagrangian equation and fractional Caputo differential theory, and simulations are carried out to analyse the influence of fractional order and other parameters of the system on the motion of the microresonator.

Introduction

Microresonator is a typical nonlinear dynamic device of micro-electro-mechanical systems (MEMS), in which the viscoelasticity plays a crucial role in its dynamics analysis. It was proved by experiment that the fractional derivative model of viscoelasticity is more accurate than the integer derivative model for soft tissue-like materials [1]. In addition to this, the fractional model has a good agreement with experimental tests on creep and relaxation, which means that fractional model may easily capture both relaxation and creep of materials [2]. And internal thermal damping of fractional model for viscoelastic materials is quite different from that of integer model [3].

In this paper, based on Rayleigh-Ritz method, Lagrangian equation and fractional Caputo differential theory, the fractional differential dynamics model of a microresonator is established with the feature of accurately describing the viscoelastic properties and internal thermal damping of materials. In the model, instead of the terms $\ddot{x}$ and $\dot{x}$ in integer differential equation, the $D^{p_1}x$ and $D^{p_2}x$ represent respectively fractional differentials, $p_1 \in (1,2)$, $p_2 \in (0,1)$,which are non-integer. The predictor-corrector method is employed to carry out the simulations to analyse influence of system parameters on the motion of the system.

Results and discussion

Figure 1 depicts the bifurcations of the system with respect to the fractional order p_1 and p_2. It is shown that the both fractional orders have obvious influence on the motion of the system. The influence of fractional order $p_1$is more significant than the fractional order p_2. This figure also indicates that chaos can be suppressed by changing the viscoelastic and internal damping of the material. In addition to this, the results of this study shows that the amplitude and frequency of excitation voltage also have certain influence on the dynamic characteristics of the system. And with the variation of excitation amplitude and frequency, the system shows more complex dynamics behavior.

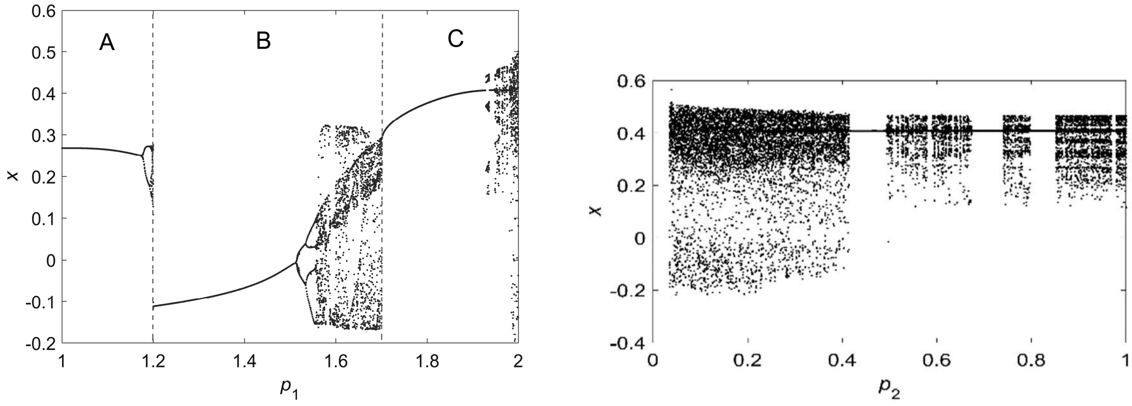

Figure 1: The bifurcation diagram versus p_1 and p_2.

References

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