# Nonlinear vibration of an inextensible rotating beam

**Lokanna Hoskoti*** and Mahesh M. Sucheendran*

*Mechanical and Aerospace Engineering, Indian Institute of Technology Hyderabad, India - 502284

**Abstract**. An inextensible beam model is developed to study the nonlinear vibration of a rotating cantilever beam undergoing large deformation. General equations of motion governing the coupled axial-bending deformation are derived, and then inextensionality constraint is utilized to reduce two equations into one. The axial strain on the neutral fiber is decomposed into static and dynamic components and the extension due to the dynamic part is assumed to be zero. Nevertheless, the static strain, which is obtained from the solution of the steady-state axial displacement equation, is retained in the equation governing the planar motion of the beam. The method of multiple scales is adopted to derive the expression for frequency response and modulation equations for the first vibration mode. The results show that the first mode exhibits a hardening type of nonlinearity for lower rotation speeds and softens at higher rotational speeds.

## Introduction

Rotating structures have many applications, such as wind turbines or helicopter blades, blades in gas turbine engines or turbo propeller blades in mechanical or aerospace engineering. Numerous authors have published papers on the dynamic analysis of a rotating blade idealized using simplified one-dimensional analytical models based on Euler-Bernoulli and Timoshenko beam theories with varying levels of complexity in geometry [1, 2]. A straight beam attached to a hub, rotating at an angular velocity, undergoing large deformation is considered in the present study. An inextensible model for a rotating beam under the harmonic excitation of frequency, $\omega$, and magnitude, $F$, can be derived using the inextensibility constraint as [3]

$$\begin{aligned}\ddot{w} &+ \left\{w''' + w'^2 w''' + w' w''^2 - (w' e_s)'' - w''' e_s\right\}' + \frac{1}{2}\left\{w' \int \left(\frac{\partial}{\partial t^2} \int w'^2 dx\right) dx\right\}' \\ &- \Omega^2 \left\{w' \int_x^1 \int_0^x \left(\frac{1}{2} w'^2 + e_s\right) dx dx - \left(w'(1 - e_s) + \frac{1}{2} w'^3\right) \int_x^1 (r + x) dx\right\}' = F \sin \omega t.\end{aligned} \tag{1}$$

Where, $w$ and $r$ are transverse displacement and hub radius nondimensionalized using the beam length. $\Omega$ is angular speed non-dimensionalised using the time, $t = \sqrt{\frac{\rho A L^4}{EI}}$

## Results and discussion

The results are presented for the beam with slenderness ratio, $\sqrt{AL^2/I} = 693$, hub ratio, $r = 0.1$. The beam is made of homogeneous material with Young's modulus $E = 104$ GPa and density $\rho = 4400\ \mathrm{kg/m^3}$ (order of magnitude of a Titanium alloy) [4]. The backbone curves of the fundamental mode of a rotating beam are plotted in Fig. 1. Each point in the backbone curve is the peak value of the frequency response curve at a given value of rotation speed and various magnitudes of excitation.

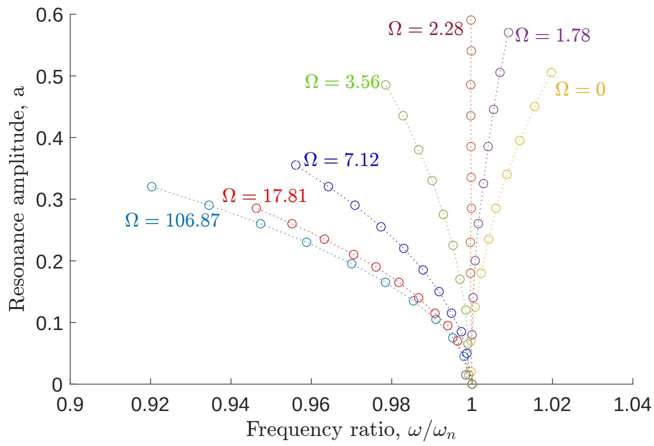


Figure 1: Backbone curves of the fundamental mode.

It is found that the rotation-induced nonlinearity significantly affects the beam's nonlinear dynamics. There exists a critical rotation speed above which the nonlinearity of the first mode becomes softening, and it is hardening below.

## References

[1] Hodges D., Rutkowski M. (1981) Free-vibration analysis of rotating beams by a variable-order finite element method, *AIAA J.* **19**(11):1459-1466.

[2] Yang J., Jiang L., Chen D., (2004) Dynamic modelling and control of a rotating Euler–Bernoulli beam, *J. Sound Vib.* **274**: 863-875.

[3] González-Carbajal J., Rincón-Casado A., García-Vallejo D., Domínguez J., (2022) Nonlinear solutions for the steady state oscillations of a clamped–free rotating beam, *Eur. J. Mech. A/Solids.* **91** : 104413.

[4] Thomas O., Sénéchal A., Deü J.-F., (2016) Hardening/softening behavior and reduced order modeling of nonlinear vibrations of rotating cantilever beams, *Nonlinear Dyn.* **86**:1293-1318.