Resonant Tunneling in a Frequency Modulated Classical Double Well Oscillator

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Abstract. We consider the parametric double well oscillator with the equation of motion $\ddot{x} - \omega^2(1+\epsilon\cos(\Omega t)) + \lambda x^3 = 0$. A particle is initially placed at rest at a minimum of one of the wells. In this work, we study the tunnelling of the particle from one well to the other by using a periodic modulation of the well depth. We find that for $\Omega \approx \sqrt{2}\omega$, the particle can tunnel out of the well for an extremely small value of ϵ. This is the primary resonant escape. We study the escape regions in the $\Omega - \epsilon$ plane and find that the boundary of the primary resonance has a fractal character.

Introduction

A shift of co-ordinates to the position of the well at $x = \omega/\sqrt{\lambda}$ casts the dynamics in the form $\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + 2\omega^2(1 - \frac{\epsilon}{2}\cos(\Omega t))y + 3\sqrt{\lambda}\omega y^2 + \lambda y^3 = \frac{\epsilon\omega^3}{\sqrt{\lambda}}\cos(\Omega t)$. An equivalent linearization at $\mathcal{O}(\lambda)$ changes the oscillation frequency from $\sqrt{2}\omega$ to $\sqrt{2}\omega\left[1 - \frac{9\lambda}{16\omega^2}A^2\right]$, where A is the amplitude of the oscillation [1]. The primary resonance in this problem comes when the driving frequency equals the 'natural frequency' [2]. We use the linearized system to estimate the critical ϵ for the escape from the well as $\epsilon_c \approx \mathcal{O}(10^{-2})$. The numerical analysis shows the strong resonant response occurring for Ω slightly below $\sqrt{2}\omega$ as shown above and $\epsilon_c \approx 0.03$. Further, we see that the boundary of the 'resonant region' clearly resembles a fractal curve. The responses at the higher harmonics and for the parametric resonance are less spectacular, however, they produce rather unexpected stability zones, as shown in Figs. 1(a) and 1(b).

Results and discussion

We use the fourth-order RK method in order to solve the second-order differential equation. Initially, at $t = 0$, the particle is placed at one minima of the double well potential. We track the position of the particle by exploring the $\Omega - \epsilon$ parameter space. As shown in Fig. (1), shaded regions represent the particle escapes from the initial well. Note that for a small value of ϵ at the resonance forcing frequency, particle from one well can tunnel to the other well. We observe a fractal behaviour at one boundary, whereas the other boundary is linear with ϵ. The escape-confined picture of the particle from the well is unaltered for any ω and λ in $\tilde{\Omega} - \epsilon$ space

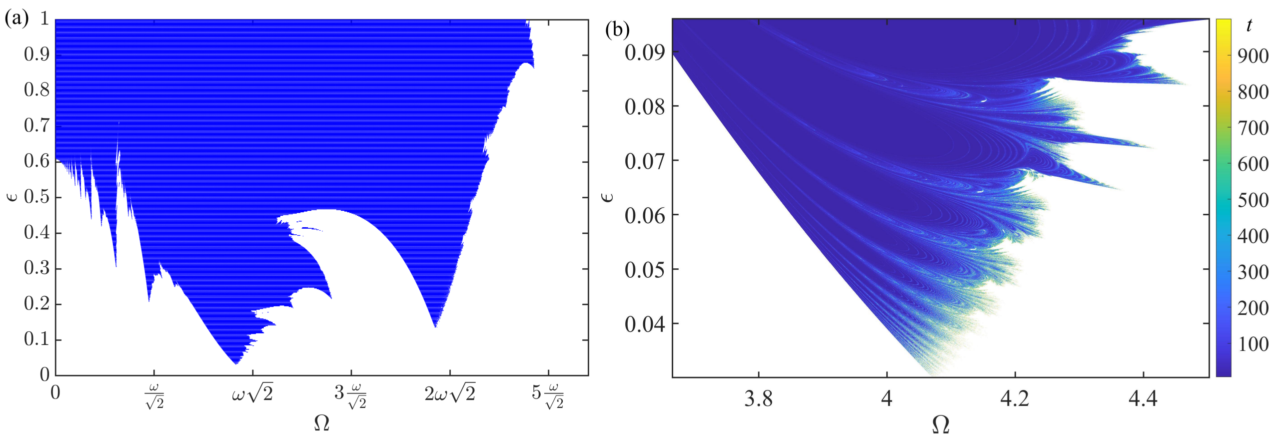

Figure 1: Solution of the parametric double well oscillator in the $\Omega - \epsilon$ parametric space for $\omega = \pi$, where, $\lambda = 0.01$, initial position $y_0 = \omega/\sqrt{\lambda}$, and initial velocity $\dot{y}_0 = 0$. Shaded regions resemble the tunnelling occurrence region, and the white region corresponds to non-escape regions from the initial well. The colourmap in Fig (b) shows the escaping time t from the initial well. The dark region corresponds to early escape, whereas the lighter region infers a delay escape from its initial well.

(normalized $\tilde{\Omega} = \Omega/\omega$), given that the particle is initially situated at the minima and initial velocity is zero. However, we report a slightly varying initial position (with zero velocity) of the particle drastically changes the escape-confined phase space.

References

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