State estimation of time-fractional reaction-diffusion SEIR model for COVID-19 with mobile sensors

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Abstract. To know the state information of the spread of diseases, such as COVID-19 is of great importance as verified by the recent pandemic. In this paper, we aim to investigate the state estimation of the spatial-temporal propagation of COVID-19 based on a time-fractional reaction-diffusion susceptible-exposed-infected-recovered (SEIR) model. For this purpose, we first regard a finite number of static sensors as mobile sensors based on the continuity of the state by traveling between static sensors and supposing that at each given time only one of the static sensors in each line is active. Next, we design an extended Luenberger-type observer that contains a state estimator and the guidance of mobile sensors to ensure the desired state estimation requirements. A numerical example is finally presented to illustrate the accuracy and the promising of our proposed state estimation strategy.

Introduction

To know the state information of epidemic system, which is required for monitoring or designing feedback control is a fundamental issue in epidemiology. However, owing to the economic or physical constraints, this is not easy to be realized. For this problem, based on system's properties and in the spirit of [1], considerable attention has been attracted on using mobile sensors to construct a Luenberger-type observer to estimate the state information of system (see e.g.[2]). This is due to the fact that mobile sensors can contain additional degrees of the freedom hence being more powerful to observing the studied system than that with non-mobile sensors. Then, in this paper, based on our previous work on time-fractional reaction-diffusion SIR epidemic system to model the spatial-temporal propagation of infection [4], we discuss the state estimation problems of the following time-fractional reaction-diffusion SEIR model

$$\begin{cases} {}_0^C D_t^\alpha S(x,t) = d_1 \sum_{i=1}^{n} \frac{\partial^2 S(x,t)}{\partial x_i^2} - \beta(x,t) I(x,t) \frac{S(x,t)}{N}, \\ {}_0^C D_t^\alpha E(x,t) = d_2 \sum_{i=1}^{n} \frac{\partial^2 E(x,t)}{\partial x_i^2} + \beta(x,t) I(x,t) \frac{S(x,t)}{N} - \gamma(x,t) E(x,t), \\ {}_0^C D_t^\alpha I(x,t) = d_3 \sum_{i=1}^{n} \frac{\partial^2 I(x,t)}{\partial x_i^2} + \gamma(x,t) E(x,t) - \kappa(x,t) I(x,t), \\ {}_0^C D_t^\alpha R(x,t) = d_4 \sum_{i=1}^{n} \frac{\partial^2 R(x,t)}{\partial x_i^2} + \kappa(x,t) I(x,t) \end{cases} \quad (x,t) \in \Omega \times [0,T] \tag{1}$$

under the output function obtained by $m-$mobile sensors

$$I^*(t) = \begin{pmatrix} I_1^*(t) \\ \vdots \\ I_m^*(t) \end{pmatrix} = \begin{pmatrix} \int_0^L c(x; x_1(t)) I(x,t) dx \\ \vdots \\ \int_0^L c(x; x_m(t)) I(x,t) dx \end{pmatrix}. \tag{2}$$

Here $\Omega \subseteq \mathbf{R}^n$ is a bounded domain with smooth boundary, ${}_0^C D_t^\alpha$, $\alpha \in (0,1]$ denotes the Caputo fractional derivative and the meanings of rest symbols will be specified in our full-length paper.

In what follows, we aim to propose an approach on using the output (2) to estimate the state of model (1).

Results and discussion

The goal of this paper is to investigate the state estimation problem of system (1) based on the measurements obtained by Eq.(2). For this, we extend our previous results in [2] and design an extended Luenberger-type observer that contains a state estimator and the guidance of mobile sensors to ensure the desired state estimation requirements using rigorous mathematical analysis. To illustrate the effectiveness of our main results, we shall present a numerical illustration by generalizing our previous works in [4] where a numerical example on the control of time-fractional reaction-diffusion SIR epidemic system is presented and in [2] where a finite number of mobile zone sensors is used to estimate the state of semilinear time-fractional diffusion systems.

References

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