Entrainment in Self-Excited Filippov Systems

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Abstract. This work investigates entrainment in a self-excited discontinuous system of Filippov type. In terms of the Poincare winding numbers, the presence of 1:*m* and *n*:*m* entrainment regions are reported. The existence of Devil's staircase is demonstrated numerically. It is shown that quasi-periodic orbits generate invariant polygons on the Poincare section, whose vertices function as natural condensation points for the generation of entrained *m*-periodic solutions.

Introduction

Entrainment in continuous, smooth systems has been studied extensively [1]. But, comparable emphasis does not seem to have been given for entrainment of discontinuous systems [2]. Here, we examine the mechanism of higher order entrainment in a 1 DoF self-excited Filippov system. The importance of invariant polygons in this context is emphasised.

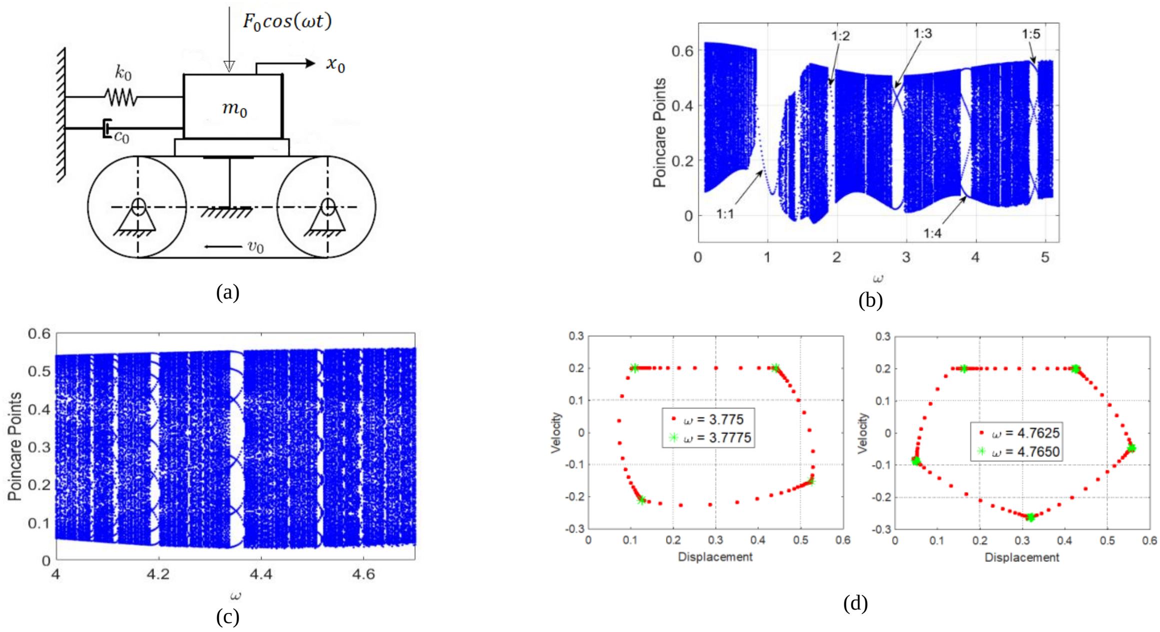

Figure 1: (a) 1 DoF Filippov model, (b) Bifurcation diagram, (c) Region between 1:4 and 1:5 windows, (d) Invariant polygons corresponding to quasi-periodic orbits and Poincare points of periodic orbits generated on their vertices.

Results and discussion

Fig. 1(a) shows the model with mass m_0 placed on a belt with uniform speed v_0. k_0 and c_0 are stiffness and damping. x_0 is the horizontal displacement. Normal excitation $F_0 cos(\omega t)$ is coupled to the horizontal motion through Stribeck frictional force between mass and the belt. The non-dimensional equation of motion can be written as $\ddot{x} + 2\xi\dot{x} + x + (1 + F_N cos(\omega t))(\mu_s sgn(\dot{x} - v_b) - k_1(\dot{x} - v_b) + k_3(\dot{x} - v_b)^3 = 0$.

Fig. 1(b) shows the bifurcation diagram with external frequency ω as parameter. In terms of the Poincare winding numbers [1], 1:1 and various 1:*m* entrainment regions are clearly observed. In Fig. 1(c) showing the enlarged view of the region between 1:4 and 1:5 entrainment windows, smaller windows of higher order *n*:*m* entrainments are visible. This points towards the presence of Devil's staircase [1] in the model. Fig. 1(d) shows the transition from quasi-periodic to *m*-periodic orbits on the Poincare section. It is seen that the quasi-periodic orbits correspond to *m*-sided invariant polygons [3]. *m*-periodic entrained solutions are born by condensation of these polygons, with their vertices acting as natural places of condensation.

References

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