Properties of a two parameter logistic map with delay

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Abstract. Logistic maps with delay (memory) have their origin in the effort [1] to discretize the integro-differential equations of non-equilibrium statistical mechanics. Buchner and Zebrowsky [2] studied it as a generic system. Their relevance in the field of economics has been established by Tarasova and Tarasov [3]. In this work , we point out that a class of infectious disease (eg. COVID-19) provide a source of a two parameter logistic map with delay. We analyze these maps to look at their fixed points, their stabilities and their transition to chaotic dynamics.

Introduction

We focus on a class of highly infectious disease where the infected person does not show any symptoms for the first few days. Such persons will be inadvertent spreaders of the disease for a number(τ) of days before they are quarantined. Following the differential equation approach [4], the rate of change of the number of infected people I_n on the n-th day will be proportional to $I_n - I_{n-\tau}$ and to the fraction of susceptible people which is $1 - I_n/N = 1 - X_n$. Assuming the total population N consists of only the infected and the uninfected but susceptible and a few of infected people are quarantined because of secondary factors we arrive at the model dynamics

$$X_{n+1} = \alpha X_n + \beta(X_N - X_{N-\tau})(1 - X_N) \tag{1}$$

This is our two parameter logistic map with delay. It has only one fixed point $X^* = 0$.The instability condition for $\tau = 1$ and $\tau = 2$ are given by:

$$\text{Instability region for } (\tau = 1) = \{(\alpha > 1) \cup (\beta > 1) \cup (\alpha + 2\beta + 1 < 0)\} \tag{2}$$

$$\text{Instability region for } (\tau = 2) = \{(\alpha > 1) \cup (\alpha < -1) \cup (2\beta^2 + \alpha\beta > 1)\} \tag{3}$$

Period doubling and onset of chaos with some variations is frequently seen when $\tau = 1$. For $\tau = 2$, we find extended period -3 regions. Alternating chaotic and periodic zones are common.

Results and Discussion

The thing to note in Fig 1.a ($\tau = 1$) is the complication following the onset of period 4. One sees the emergence of two chaotic bands and the re-emergence of the period 4 state and eventually the onset of chaos over an extended interval. In Fig 1.b ($\tau = 2$), the instability of the fixed point leads to a patterned response between two unstable fixed points characterized by avoided zones (white traces) and then the two unstable fixed points give way to a stable period three state which survives over an extended range of parameter space.

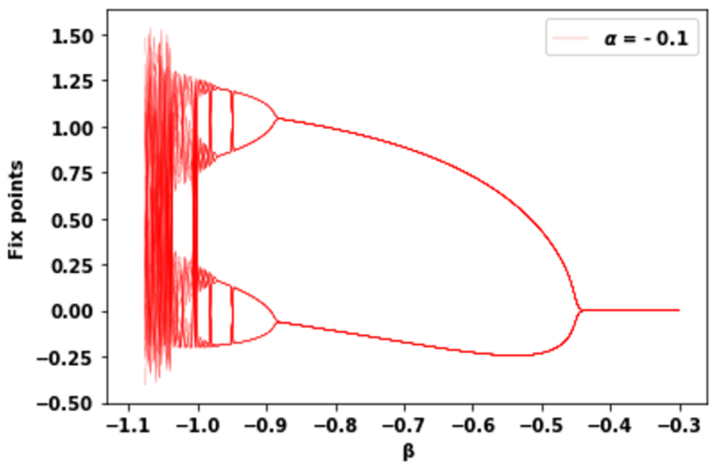

Fig 1.a: $\tau = 1$, $\alpha = -0.1$

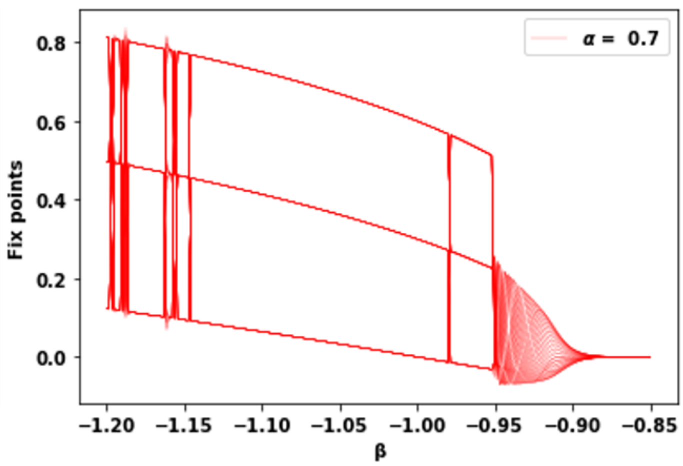

Fig 1.b: $\tau = 2$, $\alpha = 0.7$

References

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