Evaluation of Lie group Integration for Simulation of Rigid Body Systems

Stefan Holzinger and Johannes Gerstmayr
Department of Mechatronics, University of Innsbruck, Innsbruck, Austria

Abstract. As commonly known, direct time integration of the kinematic equations of rigid bodies modeled with three rotation parameters is impossible in the general case due to singular points. Common workarounds are Euler parameters, either with unorthodox normalization or by switching to differential algebraic equations. More recent approaches use Lie group integration methods, as they allow a singularity-free integration of spatial rotations. So far, however, few studies have addressed whether Lie group integration methods are more accurate and efficient compared to formulations based on Euler parameters or Euler angles, which could be a crucial aspect for the decision to extend an existing multibody code with Lie group integration methods. In this paper, we compare several explicit and implicit Lie group integration methods in terms of accuracy and computational efficiency with formulations based on Euler parameters and Euler angles using several non-linear, typical rigid body systems.

Introduction

Rigid bodies are commonly used to model complex mechanical systems. As rigid bodies have six degrees of freedom, it is obvious to use three parameters to model translations and three to model rotations. Modelling spatial rotations with three rotation parameters (RP) is problematic, as there is no singularity-free representation of spatial rotations with three RP. Formulations based on three RP apply for example reparameterization strategies [1], while state of the art formulations use Euler parameters (unit quaternions) as RP to avoid singularities [2]. More recent approaches use Lie group integration methods [3], as they not only enable a representation of spatial rotations in a setting that is free of singularities, they also allow to use three RP for modelling multibody systems [4] and exhibit favorable properties for the simulation of multibody systems. However, Lie group integration methods are inherently coordinate-free, which makes them incompatible with multibody simulation codes that are based on RP. Apart from the aspect of extensibility of an existing multibody code with Lie group integration methods, the objective of this paper is to determine, whether Lie group integration methods are more accurate and efficient compared to conventional approaches. So far, few studies have addressed the latter question, despite the fact that its answer could be crucial for the decision to extend an existing multibody code with Lie group integration methods. Therefore, in this paper, the accuracy and computational efficiency of explicit and implicit Lie group integration methods is compared to Euler angle and Euler parameter based formulations using several non-linear rigid body systems.

Results and Discussion

It is found, that explicit Lie group integration methods outperform formulations based on RP in terms of accuarcy and computational efficiency in most of the considered rigid body systems. Especially for systems with high rotational speeds, explicit Lie group integration methods turn out to be more accurate than a formulation based on Euler angles while the computational efficiency is almost the same, as exemplarily shown in Fig. 1.

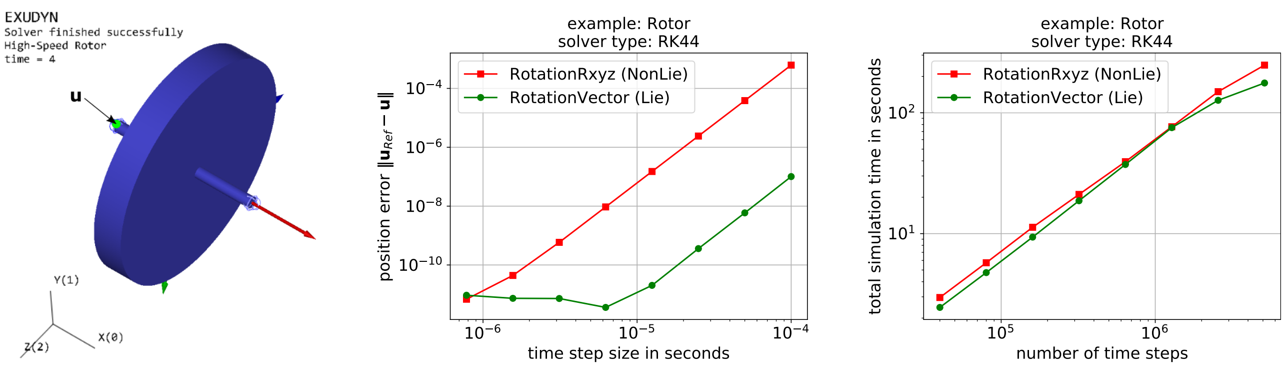

Figure 1: *Left*: Schematic representation of a flexibly mounted rotor rotating at 30000 rpm; *Center*: Convergence of the position error of the point $\mathbf{u}$ located at the rotor at the position of the left support; *Right*: Total simulation time; The abbreviation 'RotationRxyz (NonLie)' marks a formulation based on Euler angles and 'RotationVector (Lie)' marks a Lie group integration method which uses the rotation vector to model spatial rotations, cf. [4]. Both methods, use an explicit 4th-order Runge-Kutta method for time integration.

References

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