Stability analysis for multibody systems subject to bilateral motion constraints

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Abstract. A stability analysis is performed for a class of mechanical systems, with multiple bodies subject to equality motion constraints. This analysis is based on an appropriate set of equations of motion, which are expressed in terms of the original coordinates as a system of strongly nonlinear second order ordinary differential equations. The results clarify certain critical issues associated with classical numerical methodologies, like constraint violation and gradual drift from the exact response.

Introduction

Dynamics of multibody mechanical systems involving motion constraints remains a challenging problem in several technical areas of large importance, like automotive, railway, marine and aerospace structures. Usually, the equations of motion of such systems are presented as a set of differential-algebraic equations (DAEs) of high index. Since the treatment of these equations is a delicate task [1], much research effort has been devoted to the subject. Essentially, all the previous efforts are based on application of index reduction or coordinate partitioning techniques. In contrast, the equations of motion employed in the present work are a coupled set of second order nonlinear ordinary differential equations (ODEs) [2]. This is achieved by combining some fundamental concepts of Analytical Dynamics and Differential Geometry [3]. These advantages are exploited in the present work, where a stability analysis is performed based on this new set of equations of motion. This analysis is also essential for investigating the stability of various numerical integration schemes applied to the numerical discretization of the equations of motion. In contrast to previous studies on the subject, all the eigenvalues of the linearized system are bounded and are given a specific physical meaning.

Results and discussion

The class of mechanical systems examined is subject to k motion constraints with general form

$$A(\underline{q})\underline{v} = \underline{0}\,, \tag{1}$$

where $\underline{q}$ and $\underline{v}$ represent the n generalized coordinates and velocities of the system, respectively and A is a known $k \times n$ matrix. Then, by adopting the classical Analytical Dynamics framework [3], the equations of motion are derived as a set of $n+k$ second order ODEs for the $n+k$ unknowns $\underline{q}$ and $\underline{\lambda}$, where $\underline{\lambda}$ are the Lagrange multipliers. For stability analysis, appropriate linearization leads to an eigenvalue problem with form

$$\begin{bmatrix} K-\omega^2 M & -A^T(\bar{K}-\omega^2\bar{M}) \\ -(\bar{K}-\omega^2\bar{M})A & 0 \end{bmatrix} \begin{pmatrix} \hat{\underline{q}} \\ \hat{\underline{\lambda}} \end{pmatrix} = \underline{0}\,, \tag{2}$$

where the diagonal matrices $\bar{M}$, $\bar{C}$ and $\bar{K}$ are fully determined by the motion constraints [2,3]. This problem is shown to possess $m = n-k$ simple eigenvalues, coinciding with those obtained after eliminating the motion constraints, plus a set of k double eigenvalues (with geometric multiplicity 1), related to the constraints. The results shed light on certain critical issues, like the constraint violation and the gradual drift from the exact response associated with classical numerical methods. Some typical results are presented in Fig. 1.

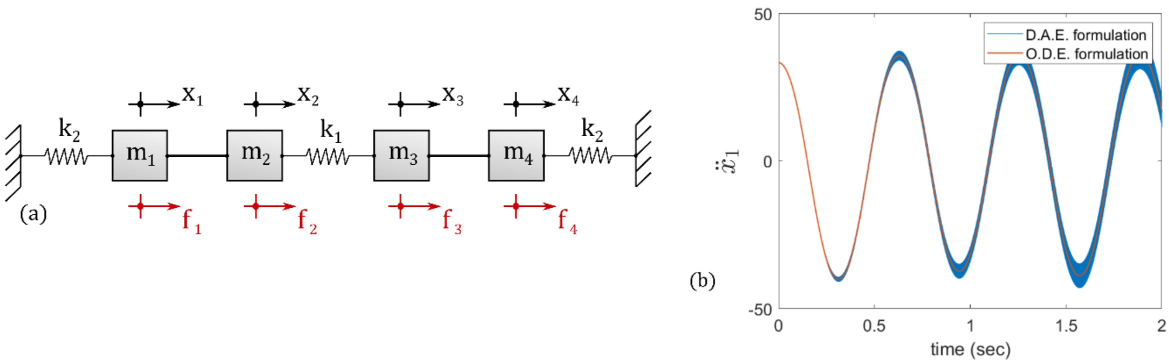

Figure 1: (a) An oscillator model and (b) numerical results (for acceleration $\ddot{x}_1$) verifying the instability of the DAE model.

References

[1] Geradin, M., Cardona, A. (2001) Flexible Multibody Dynamics. John Wiley & Sons, NY.

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