Globally-evolving-based generalized density evolution equation for multi-dimensional systems enforced by Poisson white noise

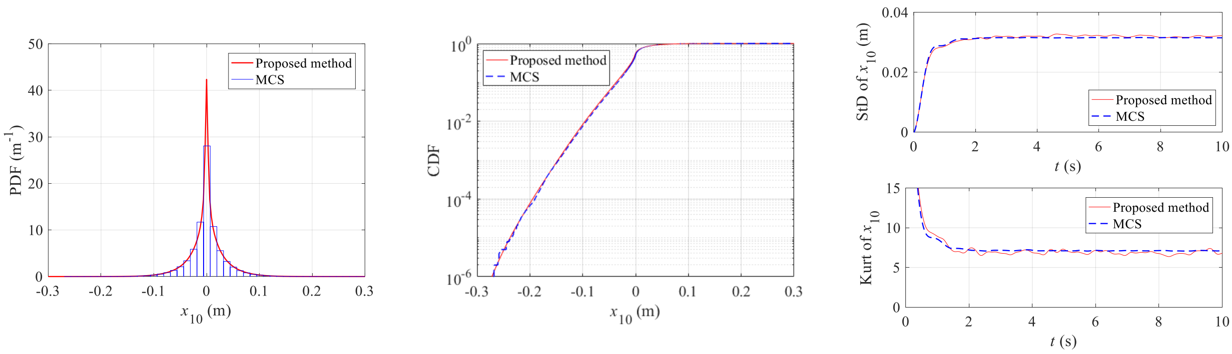
Jian-Bing Chen* and **Meng-Ze Lyu***

College of Civil Engineering, Tongji University, Shanghai, China

Abstract. Multi-dimensional stochastic dynamical systems enforced by Poisson white noise have a wide range of applications in science and engineering fields. In the present paper, a partial differential integral equation governing the probability density function (PDF) of anyone single component of interest for a multi-dimensional system enforced by Poisson white noise is established, called as the globally-evolving-based generalized density evolution equation (GE-GDEE). The effective drift coefficient in the GE-GDEE can be identified numerically, and then solving the GE-GDEE yields the solution of transient PDF of the quantity of interest. A numerical example is illustrated to verify the efficiency and accuracy of the proposed method.

Introduction

The responses of systems enforced by Poisson white noise are path-discontinuous and non-Gaussian, which has long been one of the significant challenges in stochastic dynamics [1]. Exact numerical solution for Poisson-driven single-degree-of-freedom (SDOF) systems can be given via various methods [2]. However, there is no effective general treatment for multi-degree-of-freedom (MDOF) problems except Monte Carlo simulation (MCS). Recently, a unified formalism of GE-GDEE was developed for generic path-continuous non-Markovian processes [3]. In the present paper, the GE-GDEE methodology is extended to tackle path-discontinuous processes. In the cases under Poisson excitations, the responses of interest is usually path-discontinuous, and its transient PDF satisfies the developed GE-GDEE, a one- or two-dimensional partial differential integral equation. The effective drift coefficient in the GE-GDEE is an undetermined conditional expectation function, and should be identified via the data from some representative deterministic dynamic analyses of original high-dimensional systems. Then, the GE-GDEE with the determined effective drift coefficient can be solved numerically to obtain the transient PDF of the quantity of interest. A 10-MOF Poisson-driven system is investigated to verify the efficiency and accuracy of the proposed GE-GDEE approach, and the PDF, cumulative distribution function (CDF), and statistic moment of the quantity of interest via the proposed method are compared with MCS involving 10^6 samples as shown in Figure 1. The numerical results demonstrate high accuracy of numerical solution of the GE-GDEE.



(a) PDF; (b) CDF; (c) Standard deviation and kurtosis.

Figure 1: Numerical results by the proposed method compared with MCS.

Results and discussion

In the present paper, a governing equation for transient PDF of quantity of interest for multi-dimensional Poisson-driven stochastic dynamical system as a partial differential integral equation, called as the GE-GDEE, is established. The numerical solution of the GE-GDEE yields the probabilistic response results with high efficiency and accuracy.

References

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