Inverse Design of Periodic and Quasi-Periodic Nonlinear Mechanical Metamaterial

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Abstract. In nonlinear ultrasonics, the interaction of a monochromatic wave with nonlinear elastic materials generates higher harmonics (*2f, 3f, ….)*, known as harmonic generation. The sensitivity of higher harmonics amplitudes due to harmonic generation and harmonic scattering towards design parameters of nonlinear metamaterials, such as widths of the nonlinear elastic layers, makes the inverse design of nonlinear metamaterials challenging. Periodic and quasi-periodic linear and nonlinear mechanical metamaterials are designed by solving inverse design problems to control nonlinear elastic waves in solids. The time-dependent inverse design problems include a forward problem implemented using the finite element method. The inverse problem is proposed as a time-dependent optimization problem and solved using the Nelder-Mead algorithm.

Introduction

Nonlinear ultrasonics has shown its effectiveness in quantifying early-stage damages such as micro-cracks, dislocation substructure, micro-voids, etc. Harmonic generation due to the interaction of monochromatic (*f*) waves with early-stage damages is demonstrated in various theoretical [1], computational [2], and experimental [3] studies. Measuring amplitudes of such higher harmonics is important as they give information about the intensity of early-stage damages. During experiments, system-generated higher harmonics are introduced due to instrumentation, such as a short pulse power amplifier that masks the damaged material's natural harmonic response [4]. Suppression of such system-generated higher harmonics like 2nd (*2f*) and 3rd (*3f*) harmonics can be achieved by designing appropriate metamaterials. The harmonic generation and scattering are highly sensitive to the widths of metallic layers present in metamaterials [1]. The nonlinearity effects of the materials used in the design of metamaterials need to be addressed. This research aims to design robust linear and nonlinear metamaterials for nonlinear ultrasonic applications through the inverse design approach. A shape optimization problem is defined for the design of layered linear, nonlinear, periodic, and quasi-periodic (Figure 1) metamaterials by simulating wave propagation studies using the finite element method.

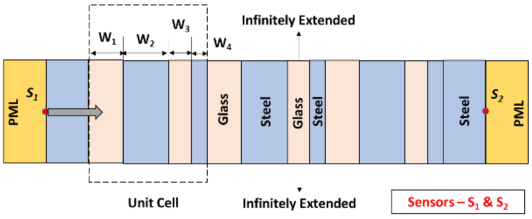
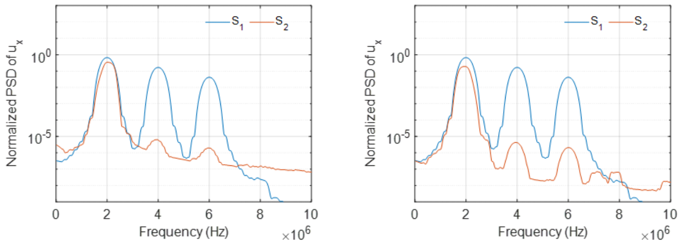


Figure 1: Schematic of quasi-periodic metamaterial.



a) b)

Figure 2: Frequency responses of the waves passing through the inversely designed periodic (a) and quasi-periodic (b) nonlinear metamaterials that suppress 2nd (*2f*) and 3rd (*3f*) harmonics as well as maintaining the amplitude of the 1st (*f*) harmonics maximum.

Results and Discussion

Capabilities of the inversely designed periodic and quasi-periodic linear and nonlinear metamaterials to control nonlinear waves are demonstrated through this novel, most realistic, and universal inverse design approach (Figure 2). Interestingly, some inversely designed linear metamaterials also show similar responses to nonlinear metamaterials. Harmonically scattered waves are trapped within the metamaterial due to the complex interplay between harmonic generation and scattering of waves between multiple nonlinear elastic layers.

References

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