An extensible double pendulum and multiple parametric resonances

Shihabul Haque*, Nilanjan Sasmal* and Jayanta K. Bhattacharjee*

**School of Physical Sciences, Indian Association for the Cultivation of Science, Jadavpur, Kolkata 700032, INDIA*

Abstract. We consider a double pendulum where the lower pendulum is an elastic one. This is a system with three natural frequencies and we find that the responses of a hard spring and a soft spring are very different with the resonance at the beat frequency being of special interest.

Introduction

Our system is a double pendulum with masses m_1 and m_2 having lengths l_1 and l_2 respectively. The lower pendulum (m_2, l_2) is elastic and has a spring constant k. The Lagrangian of the system can be written in terms of the angular displacements, θ_1 and θ_2, made with the vertical and the extension of the lower mass m_2 with respect to its equilibrium position, r. We focus on the quadratic linearity (first deviation from normal mode dynamics). The dynamics is then governed by,

$$Ml_1^2\ddot{\theta_1} + Mgl_1\theta_1 + m_2l_1(l_2+r)\ddot{\theta_2} + 2m_2l_1\dot{r}\dot{\theta_2} + kl_1(\theta_1-\theta_2)r = 0$$

$$(l_2+r)\ddot{\theta_2} + l_1\ddot{\theta_1} + 2\dot{r}\dot{\theta_2} + g\theta_2 = 0$$

$$\ddot{r} + \omega_0^2 r = l_1(\theta_1-\theta_2)\ddot{\theta_1} + l_1\dot{\theta}_1^2 + l_2\dot{\theta}_2^2 + \frac{g}{2}\theta_2^2$$

Here, $M = m_1 + m_2$. The normal mode frequencies are Ω_1, Ω_2 for the two angular modes and ω_0 for the extension mode. We can read off the existence of four parametric resonance conditions as,

$$\omega_0 = 2\Omega_1,\ 2\Omega_2,\ \Omega_1 \pm \Omega_2$$

In each case one can construct a six dimensional dynamical system following the standard procedure [1]. We show here the results for the resonance in the beat mode, $\omega_0 = \Omega_1 - \Omega_2$, and $\omega_0 = 2\Omega_2$.

Results and Discussion

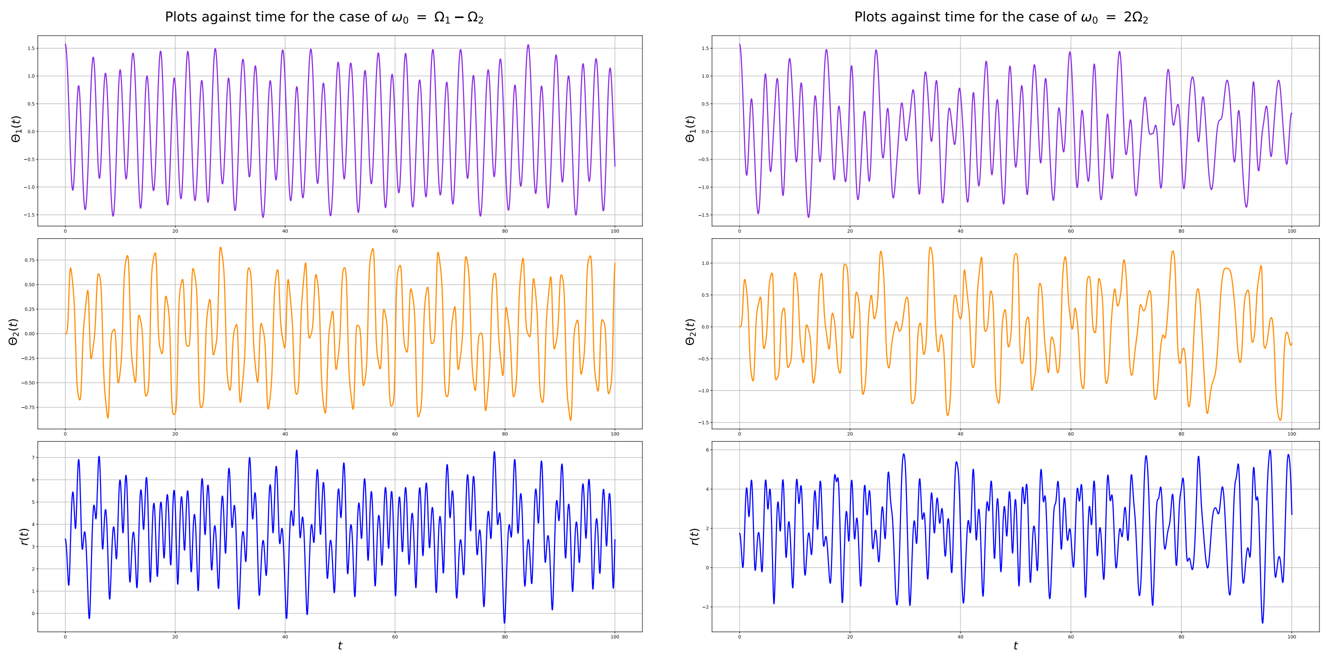

Fig: Time variation of the 3 modes for 2 different resonance conditions. The values taken for the plots are: $m_1 = 5.6\ kg$; $m_2 = 5\ kg$; $l_1 = 3\ m$; $l_2 = 3.4\ m$. *The spring constant is about* $14.69\ N/m$ *for the beat frequency and* $28.1\ N/m$ *for the other.*

We find 4 distinct parametric resonances at $\omega_0 = 2\Omega_1,\ 2\Omega_2,\ \Omega_1 \pm \Omega_2$. Of these, 2 are plotted above. The beat frequency resonance shows a marked periodicity in the variation of the first 2 modes which is particularly striking and suggests a distinct response compared to other values of the frequencies. Further work is being done to determine an approximate analytical description of the amplitude variation of the 3 modes under resonance.

References

[1] Lai H. M. (1984) On the recurrence phenomenon of a resonant spring pendulum. *Am. J. Phys.* **52**:219-223.

[2] Jia, Y., Du, S., & Seshia, A. A. (2016) Twenty-Eight Orders of Parametric Resonance in a Microelectromechanical Device for Multi-band Vibration Energy Harvesting. *Sci. Rep* **6** 30167. https://doi.org/10.1038/srep30167