Nonlinear interactions in a nonlinear time-dependent chain

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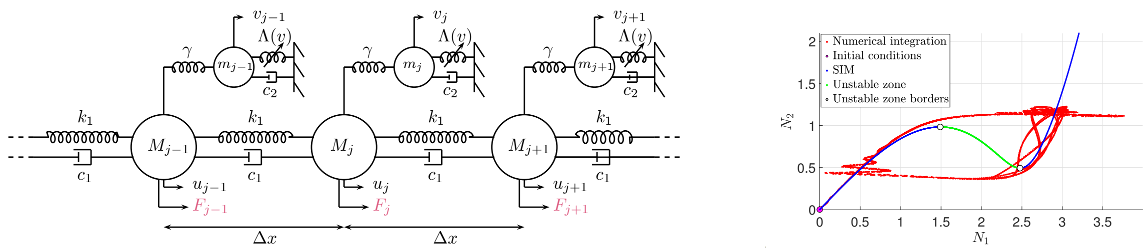
Abstract. The energy exchanges in a chain of oscillators is studied. The chain is composed by identical cells which correspond to a two degree-of-freedom (dof) systems composed by a linear oscillators linearly coupled to a grounded nonlinear oscillator with a time-dependent rigidity. The detection of the different dynamics makes it possible to predict the different regimes of the system.

Introduction

We consider a chain of coupled oscillators (see Fig. 1a) which is composed of n identical cells. A unit cell is a two dof system composed of a linear main oscillator (mass M_j) weakly coupled to a grounded nonlinear oscillator (mass m_j) [1] ($j \in [1, n]$). Two adjacent masses M_j are equally spaced at rest position with the distance Δx. We set u_j and v_j the displacements of the mass M_j and m_j, respectively and $\Lambda(v)$ as the nonlinear time-dependent restoring function. We suppose furthermore that $m_j \ll M_j$, each mass M_j is under external excitation ($F_j(t)$) and the chain is periodic.



(a) (b)

Figure 1: a) Schematic of the studied system: a chain of linearly coupled M_j, $j = 1, 2, .., L$ masses which are locally and linearly coupled to m_j masses possessing nonlinear restoring forcing functions. The M_j masses are equally spaced (Δx) at rest positive. b) Slow Invariant Manifold (SIM) (blue line) and numerical integration of governing equations (red line) for the projected chain with a constant nonlinear rigidity.

Results and discussion

We would like to predict responses of the mentioned chain around one of its modes. For this we study the projected equations on an arbitrary mode of the system for a constant and time-dependent nonlinear rigidity. Thus the model of the chain is reduced to a two dof system. The idea is to design the chain for localisation of vibratory energy [2]. For this we will detect the SIM, the fast and slow dynamics of the projected system via introducing the complex variable of Manevitch [3] and using the multiple scale method [4]. This will lead to the determination of characteristics points of the system: equilibrium and singular points corresponding to possible periodic and non-periodic regimes. As an example, Fig. 1b depicts the SIM of the projected system with "constant cubic nonlinearity" (in blue) as function of system amplitudes (N_1 and N_2). This figure is supplemented by numerical results of the forced system (in red). Due to existence of singularities, the system presents modulated response corresponding to repeated bifurcations between stable branches of the SIM. The same study is carried out on the chain with time-dependent nonlinearity which will be presented and commented upon.

To validate and show the relevance of the projection method, the numerical results of the projected chain are compared to those of the discrete chain.

References

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