Data-driven bifurcation analysis using a parameter-dependent trajectory

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Abstract. Identification of bifurcation diagrams in nonlinear systems is of great importance for resilient design and stability analysis of dynamical systems. Data-driven identification of bifurcation diagrams has a significant advantage for large dimensional and experimental systems where accurate system equations are not available. In this work, a novel forecasting approach to predict bifurcation diagrams in nonlinear systems is presented using a single trajectory of system dynamics before instabilities occur. Unlike previous techniques, the proposed method considers a varying bifurcation parameter during the same system recovery. This method is a hybrid approach that combines an asymptotic analysis provided by the method of multiple scales and a data-driven forecasting technique. Using the proposed approach allows stability analyses of nonlinear systems with limited access to experimental or surrogate data.

Introduction

Analyzing bifurcation diagrams is a significant challenge because the characterization of the post-bifurcation regime using traditional methods is a difficult task, particularly for large dimensional systems and structures. While time integration requires calibrated models and large computational costs, data-driven bifurcation analysis techniques appear as a potential alternative. In this context, Epureanu and Lim [1] proposed a technique to predict and construct bifurcation diagrams using measurements of system dynamics at several control parameters before instability occurs. This technique is based on the critical slowing down phenomenon (CSD) observed in nonlinear systems approaching certain types of instabilities including Hopf bifurcations. The forecasting technique has shown a significant advantage in predicting bifurcation diagrams such as large dimensional aeroelastic systems [2]. However, this method requires several recoveries at different constant flow speeds to predict the post-flutter dynamics. To overcome this limitation, we propose a hybrid forecasting method combining the bifurcation forecasting method and the method of multiple scales (MMS). The proposed forecasting approach uses the asymptotic analysis of the MMS to obtain a general normal form of the bifurcation for a system with general nonlinearities. The approximated normal form allows capturing the CSD as the bifurcation parameter varies during the system recovery. In consequence, the multiple recoveries requirement is leveraged.

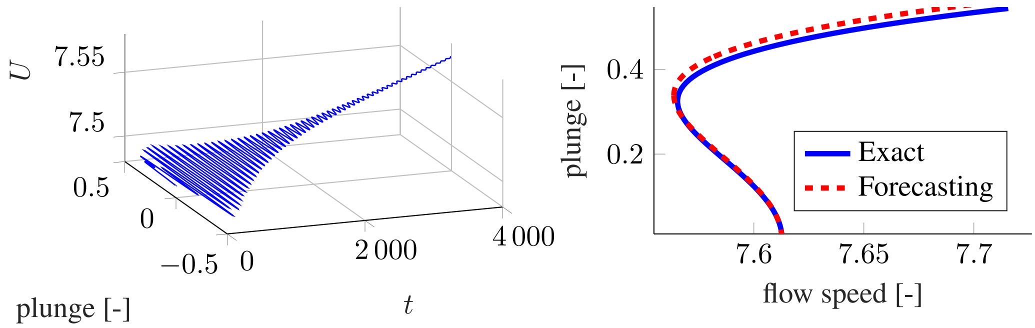

Figure 1: System recovery from perturbation with time varying flow speed (left) and exact and forecasted bifurcation diagram with one recovery forecasting for subcritical bifurcation (right)

Results and discussion

The proposed technique is applied to predict the bifurcation diagram of a typical pitch-plunge aeroelastic section exhibiting Hopf bifurcations. After applying a perturbation to the system, the flow speed is varied linearly from 97 to 99 % of the linear flutter speed to generate a trajectory. Figure 1 shows the single system recovery from perturbation (left) used to forecast the bifurcation diagram for the non-dimensional plunge (vertical displacement) for a subcritical Hopf bifurcation of the pitch-plunge airfoil (right). Results show that the approximated diagrams match the reference diagrams in both supercritical and subcritical cases despite the fact that only a single trajectory is used to predict and construct the diagrams. The proposed approach not only leverages the requirement of multiple measurements to forecast the bifurcation but also can provide early warnings of imminent instabilities as the flow speed is increased and the system is perturbed.

References

[1] Lim, J., Epureanu, B. I. (2011) Forecasting a class of bifurcations: Theory and experiment. *J. Phys. Rev. E* **83**:016203.

[2] Ghadami, A., Epureanu, B. I.. (2018) Forecasting critical points and post-critical limit cycles in nonlinear oscillatory systems using pre-critical transient responses. *Int. J. Non-Linear Mech* **101**:46–156