Some comments on Nonlinear Dynamic behaviour and Control of a Duffing 3D oscillator

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Abstract. In this work, the nonlinear dynamic behaviour of a three-dimensional Duffing oscillator was numerically investigated. The oscillator has an additional nonlinear feedback equation of state coupled to the first equation, and therefore has two parameters that determine coupling and control the 3D system. With the non-linear dynamic analysis, it was obtained using the maximum Lyapunov exponent sweeping the coupling parameters of the equations, in this analysis we observed the emergence of shrimp patterns. Bifurcation diagrams, phase maps and Poincaré maps were also used, which corroborated to determine and confirm the chaotic regions of the 3D Duffing oscillator. Once the regions were determined, a control design for chaos suppression was proposed that kept the 3D Duffing oscillator in periodic orbit, using the Optimal Linear Feedback (OLFC) control due to its high computational efficiency and SDRE (State Dependent Riccati Equation).

Introduction

An oscillator that has a lot of prominence is the Duffing oscillator, which is used to describe numerous applications. This paper aims to analyze the nonlinear dynamics of the Duffing oscillator in three dimensions as shown in Eqs. (1):

$$\begin{cases} \dot{x} = \ y + z \\ \dot{y} = \ -\gamma y + x - x^3 + f_0 \cos(\omega t) + \rho x z \\ \dot{z} = \ -xy + \beta z \end{cases} \tag{1}$$

where, x, y and z are the system state variables, γ, f_0 and ω are positive definite constants of the Classical Duffing system. The parameters ρ and β are the additional parameters that couple the equations. The aim work is to analyze the nonlinear dynamics of a 3D Duffing oscillator described by the set of equations [1, 2], and thus propose two control designs to suppress the chaotic behavior that the system presents for a given set of parameters.

Results and discussion

The State Dependent Riccati Equation control (SDRE control) and the Optimal Linear Feedback control (OLFC control) were used, and we compared that the two controllers were efficient for the chaos suppression for coupling to an orbit of the system itself, defined by ($x(t)=-0.8873+ 0.34 \cos(\omega t)+0.1037 \sin(\omega t)-0.08245 \cos(2\omega t)$ $0.03784 \sin(2\omega t)+0.01262 \cos(3\omega t)+0.01102\sin(3\omega t)$, $y(t)=-7.491\times10\text{-}5+ 0.09261 \cos(\omega t)-0.3116 \sin(\omega t)-0.0635 \cos(2\omega t)+ 0.1486 \sin(2\omega t)+0.002646 \cos(3\omega t)+0.03334\sin(3\omega t)$ and $z(t) =5.785\times10\text{-}5+ 0.0114 \cos(\omega t)-0.02837 \sin(\omega t)-0.01251 \cos(2\omega t)0.01618 \sin(2\omega t)+0.006706 \cos(3\omega t)-0.004394\sin(3\omega t)$). To obtain the orbit we consider the time series defined by Eqs. (1) and Fourier Series and the parameters ($\gamma = 0.25$, $f_0 = 0.3$, $\omega = 1.0$. $\beta = -10.4060$ and $\rho = -0.185$). With that we couple the control signals in the three variables of the system *(x,y,z)*. The very low errors in the application of the control techniques showed that both are efficient for application in the case of the 3D Duffing oscillator. In Figure (1) show the results for SDRE control applied in Eqs.(1) and Figure (2) show the results for OLFC applied in Eqs. 1.

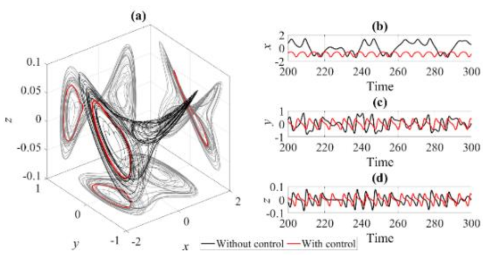

Figure 1: SDRE control applied in Eq. (1) for synchronization with orbit. Chaotic trajectory (Line black) and orbit controlled (red line) (a) Phase Portrait for $\gamma = 0.25$, $f_0 = 0.3$ and $\omega = 1.0$., (b) Time series for *x*, (c) Time series for *y* and (d) Time series for *z*.

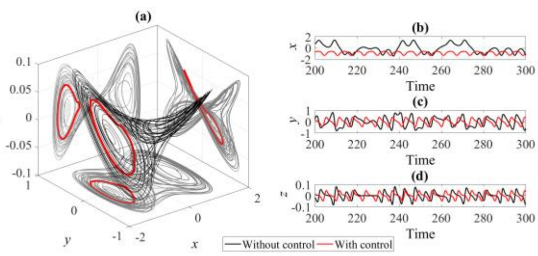

Figure 2: OLEC control applied in Eq. (1) for synchronization with orbit. Chaotic trajectory (Line black) and orbit controlled (red line) (a) Phase Portrait for $\gamma = 0.25$, $f_0 = 0.3$ and $\omega = 1.0$., (b) Time series for *x*, (c) Time series for *y* and (d) Time series for *z*.

References

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