Higher order transverse discontinuity mapping for hybrid dynamical systems

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Abstract. This work presents a higher order correction to the transverse discontinuity mapping(TDM) for accurate prediction of trajectories for piecewise-smooth(PWS) hybrid systems where degree of smoothness(DoS) is zero. To demonstrate this, a PWS system representing the simplest case of fluid-structure interaction(FSI) where the structure undergoes vortex induced vibrations(VIV) in a uniform flow is studied. Nonsmoothness manifests when a barrier is encountered obstructing the structure's motion. Investigation of this PWS FSI system show discontinuity induced bifurcations(DIBs) and chaos. The derived higher order TDM is implemented to perform a stability analysis and corresponds well with the bifurcation results. A comparison of the higher order corrections with the first order TDM shows significant improvements. The necessity of a higher order TDM to accurately predict the outcomes of a PWS impacting system is highlighted.

Introduction

PWS systems exhibit behaviours like DIBs (grazing, sliding, chattering), period adding cascades, coexistence of multiple attractors, quasi-periodicity and chaos [1]. To estimate stability of the linearized states, the behaviour of infinitesimally perturbed trajectories, during a border collision, is examined. Two nearby trajectories interact with the barrier at different instants of time, δ_is. With the predicted flight times, the perturbed paths are mapped using TDM; only valid for transversal interactions with the boundary. Therefore, it is essential to accurately predict δ_is, since the TDM is a function of δ_is and the separation between the trajectories. In general for impacting systems, the $\mathcal{O}(1)$ terms of $\delta(t)$ are not a function of the system parameters. Thus, neglecting higher order terms causes incorrect prediction of the dynamics. Moreover, for some parameters showing chaotic trajectories, the perturbed paths exponentially diverge as the system evolves. For such cases, the $\mathcal{O}(1)$ terms of δ_is might lead to inaccurate estimates of the state. Here, the time difference, δ_is and subsequently the TDM for the PWS FSI system are derived while retaining the $\mathcal{O}(2)$ terms. To demonstrate the applicability, a 4-D FSI system comprising of a structural (harmonic) oscillator ($y(t)$), undergoing VIV transverse to the cross-flow, is assessed [2]. Here, the lift force is modelled as a Van der Pol oscillator. Non-smoothness is introduced by placing a barrier in the vicinity of the structure. The structure, upon interaction with the barrier, undergoes an instantaneous reversal of velocity defined by a restitutive law with coefficient of restitution $r = 0.8$.

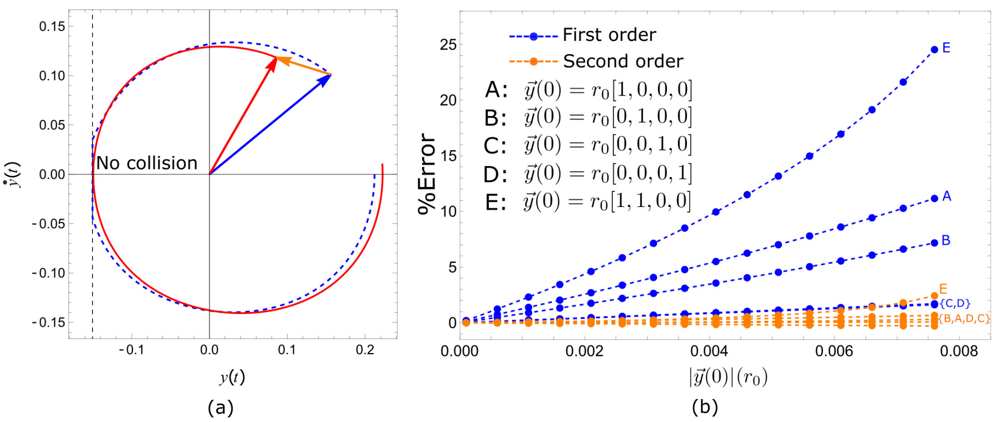

Figure 1: (a) Perturbed trajectory (in red) with initial norm of 0.01 units showing no border collision (b) % Error in δ using $\mathcal{O}(1)$ and $\mathcal{O}(2)$ with initial perturbations $\vec{y}$ lying on 4-D hyper-sphere labelled A to E normalized as r_0, shown in x axis.

Results and discussions

A comparison of % error in δ_is using $\mathcal{O}(2)$ [3] over $\mathcal{O}(1)$ reveals a significant improvement, see Fig. 1(b). The $\mathcal{O}(2)$ terms of δ_is and TDM indicate that some trajectories might miss the discontinuity boundary, see Fig. 1(a). This is contradictory to the predictions using $\mathcal{O}(1)$ and the saltation matrix, which states that an impact will occur, that leads to an incorrect prediction of the perturbed trajectories. Direct numerical simulations reveal that the $\mathcal{O}(2)$ accurately predicts the behaviour of these trajectories near the border.

References

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