Hopf Bifurcation Analysis of the BVAM Model for Electrocardiogram

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Abstract. Bio-electric activity of heart is modeled by the Barrio-Varea-Aragon-Maini (BVAM) Model. This model covers normal rhythm and several arrhythmia that lie in the chaotic region and exhibits several bifurcations, starting from a fixed-point bifurcation leading to chaotic region. Analytic solution of the BVAM model is developed in the local region to Hopf bifurcation. Center manifold reduction is applied to the governing equations to reduce the order of the system. The Method of Multiple scales is used on the center manifolds to develop normal form of the Hopf bifurcation for the center manifolds. These are then transformed back into original coordinates where the analytical solution is compared with the numerical solution.

Introduction

Heart disease is one of the leading causes of death in the world. Electrocardiograms (ECG) are time series signals used to inspect the activity of heart. There are various mathematical models that reproduce these ECG signals. A primitive model was developed by van der Pol [1] by coupling simple ordinary differential equations (ODE), which produces a signal that closely matches the sinus rhythm of ECG signal. Since then, several other models that capture normal sinus rhythm of the heart and arrhythmia [2] have been proposed. These ODEs are complex in nature and provide insight [3]. One such equation is the Barrio-Varea-Aragon-Maini (BVAM) model, which is obtained from generic reaction-diffusion system describing various patterns observed in biological and chemical systems [4, 5]. Recently this model was used for producing ECGs of normal sinus rhythm and arrhythmia [6].

In the present study, nonlinear analysis is performed on BVAM model that represents the bio-electric activity if the heart to identify bifurcation points. For one of the control parameters, Hopf bifurcation is determined and analytical solution is obtained using the Method of Multiple Scales. Normal forms of Hopf bifurcations are also determined which show the variation of amplitude against a single control parameter.

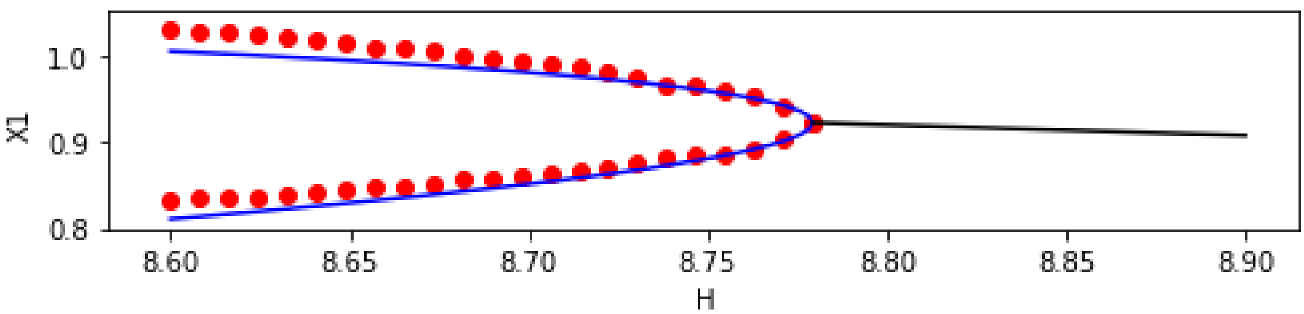

Figure 1: Amplitude Response Curve for one of the state variables x_1 against control parameter H, Stable Fixed Points [Black Solid line], Analytic Periodic Solution [Blue Solid line], Numerical Periodic Solution [Red dots]

Results and Discussion

We find two stable and one unstable fixed-points prior to Hopf bifurcation point. The two stable fixed-point solutions become unstable at Hopf bifurcation and a periodic solution emerges from the point of bifurcation for one of the state variables as shown in figure (1). Analytical solutions are obtained for these bifurcation points.

References

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