Managing parametric frequency noise using nonlinearity in a High-Q micromechanical torsion pendulum

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Abstract. We derive a nonlinear equation of motion for a micromechanical torsion pendulum and reveal that the dependence of its frequency on amplitude is a conduit for noise in a proposed "clock gravimeter" application. Using the method of multiple scales, we carry out a first order expansion capturing nonlinear effects due to the nanoribbon suspension and pendulation of the mass, express the frequency correction in terms of physical geometry and material properties, and show that the dependence on amplitude can be greatly reduced by judicious selection of device geometry.

Introduction

Gravity can be detected by tracking the frequency of a pendulum. But for microscale pendulums (e.g., Figure 1) to resolve local gravity with relative precision of 10^{-8} in a reasonable time (<1000 seconds), they must be resonantly driven to amplitudes that exceed noise by 3 to 4 orders of magnitude. Such amplitudes exceed the linear range of frequency invariant behaviour, so we must consider the parametric dependence of frequency on amplitude and its consequences.

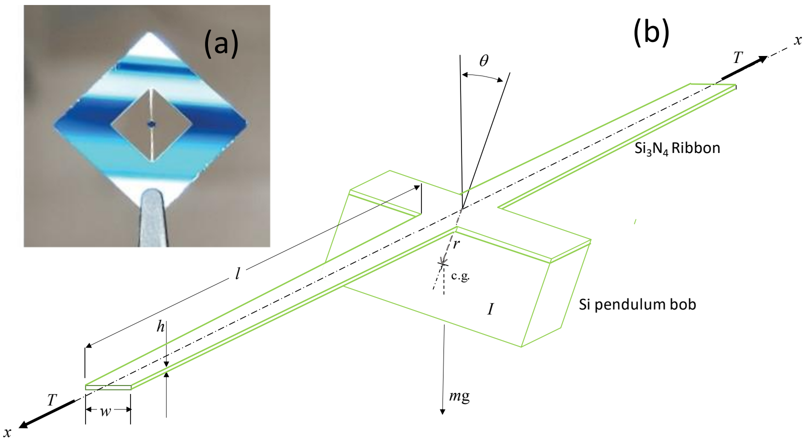

Figure 1: (a) photo of torsion pendulum device (b) schematic of torsion pendulum, formed by suspending a rectangular balance beam of mass m from a ribbon-like torsion fiber under tension, T . Angular displacement of the balance beam θ is measured using an optical lever. The center of gravity (c.g.) of the balance beam is offset a distance r from the rotation axis, resulting in a gravitational restoring torque, $\tau_g = mgr \sin\theta$ that causes the beam to pendulate

Results and discussion

Previously, we found the torsional stiffness of the device of Figure 1 has three components, one dissipative, arising from shear of the ribbon, one nominally conservative, arising from residual stress in the ribbon, and one wholly conservative due to the action of gravity on the suspended mass (e.g., pendulation). The conservative stiffness terms enabled the construction of high-Q devices (Q > 10^7) sensitive to gravity yet exceptionally small, with potential as chip-scale relative gravimeters [1]. Here, we extend the previous linear analysis to reveal the projection of amplitude noise onto the pendulum frequency. We include a nonlinear restoring torque arising from mid-plane stretching of the ribbon, finding it is a hardening spring that can be manipulated to counteract the softening character of the gravitational torque. Using the method of multiple scales [2], we carry out a first order expansion including both nonlinear effects, express the frequency correction in terms of physical geometry and material properties, and show that the dependence on amplitude can be greatly reduced by judicious selection of device geometry.

References

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