Abstract Dynamics: An alternative approach to local Lyapunov exponents in examining local unpredictability

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Abstract. This work proposes a computational approach that has its roots in the early ideas of local Lyapunov exponents, yet, it offers new perspectives toward analyzing these problems. The method of interest, namely abstract dynamics, is an indirect quantitative measure of the variations of the governing vector fields based on the principles of linear systems. The examples in this work, ranging from simple limit cycles to chaotic attractors, are indicative of the new interpretation that this new perspective can offer. The presented results can be exploited in the structure of algorithms (most prominently machine learning algorithms) that are designed to estimate the complex behavior of nonlinear systems, even chaotic attractors, within their horizon of predictability.

Introduction

An interesting idea in the analysis of nonlinear dynamics that stems from the case of linear systems is the notion of Lyapunov exponents. Lyapunov exponents are a measure of the exponential divergence of the trajectory from its original path upon experiencing an infinitesimal disturbance [1]. The Lyapunov exponents are characteristics of the attractor and determine the general expansion or attraction of the motion in the long run and are global properties that are independent of the trajectory if the dynamics is ergodic [2]. In sharp contrast, the idea of local Lyapunov exponents (LLEs) attempts to truncate the global approach of the Lyapunov exponents to the case of the divergence of infinitesimally perturbed trajectories in a finite (and mostly short-term) time interval. This idea is of significant importance when the short-term behavior of nonlinear dynamics is of interest. For many chaotic systems, such as the models of climate and economic dynamics, long-term prediction is proven to be impossible and frankly unnecessary but short-term predictions play a critical part [3].

The contribution of this work is the observation of patterns, and more specifically, dependencies in the eigenvalues of the Jacobian matrix when evaluated along the trajectory of the system. This observation stems from the fact that contrary to previous works, the time evolution of the eigenvalues is not the primary target but their relative behavior is examined. It is then observed that in nearly all the examples, there are areas, in the space of these eigenvalues, called the *abstract dynamics* space, where the eigenvalues vary linearly. Additionally, the one-to-one mappings of the eigenvalues to the trajectory of the state variables indicate an area-to-area mapping.

Results and Discussions

The results of this study indicate the existence of a level of order, although limited, in the response of certain chaotic systems that was not known before. The identification of this limited order can ease the local estimation process of chaotic attractors and may, in the most optimistic case, enable the obtainment of their analytical responses. Moreover, the results of this paper demonstrate the fading of the transient response of certain dynamical entities when the system is observed through this methodology. Inexorably, this can advance and simplify the identification process of numerous systems using raw numerical data. The scope of the applications are not limited to these two examples and more will be discussed in the full manuscript. Figure 1 demonstrates the abstract dynamics of the Lorenz attractor and it can be observed that the motion moves on a *cardinal line* for an extended period which has several implications.

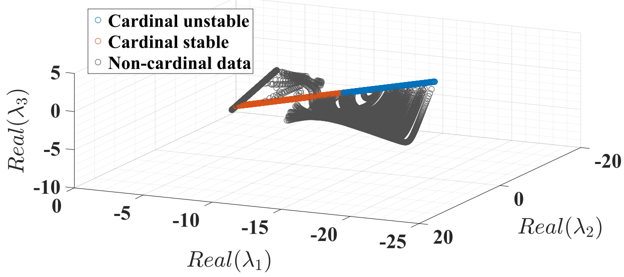

Figure 1: Abstract dynamics of the Lorenz attractor indicating a level of order in certain portions of the motion.

References

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