An Alternative Approach to Model Milling Dynamics

Kaidong Chen[a,b], He Zhang[a], Nathan van de Wouw[b] and Emmanuel Detournay[a]
[a]*Department of Civil, Environmental and Geo-Engineering, University of Minnesota, MN, USA*
[b]*Department of Mechanical Engineering, Eindhoven University of Technology, Eindhoven, The Netherlands*

Abstract. In models for the dynamics of the milling process, the characterization of the chip thickness plays a central role. In this abstract, we develop an alternative model for the computation of the chip thickness. The chip thickness is computed using (i) a surface function describing the evolution of the milled surface and (ii) the information about the shape of the workpiece. Combining a partial differential equation (PDE) governing the surface function with the ordinary differential equations (ODE) governing the tool dynamics, a mixed PDE-ODE model formulation is proposed to describe the milling process dynamics. Compared with the classical delay differential equation (DDE) formulation for milling dynamics, this novel formulation is more accurate because less assumptions have been made to compute the chip thickness. Case studies show that the two formulations generally agree well with each other. However, a sizable difference on predicted limit cycles, characterizing the periodic tool motion, occurs when the axial depth of cut is relatively large. This PDE-ODE formulation brings a novel perspective for analyzing milling processes as well as a means to assess the validity of models that based on DDE formulation.

Introduction

Dynamic models of the milling process can be classified into two classes [1]. Class I models only consider the movement of the tool and formulate the regenerative effect in terms of the current and specific previous positions of the tool. The majority of Class I models are formulated mathematically in terms of a delay differential equation (DDE) [2, 6]. In contrast, the Class II models simulate both the tool motion and the machined surface around the tool [1]; they provide a more direct way to compute the chip thickness. However, the published Class II models focus on the numerical discretization of the surface without an explicit mathematical framework, unlike the DDE formulation for Class I models. The aim of this work is to formulate the governing equations underpinning the Class II models. Inspired by the partial differential equation (PDE) approach [3, 4] recently developed to describe the evolution of the machined surface in the turning process, the authors formulated a PDE that governs the evolution of the machined surface in milling [5]. Here this PDE is combined with a system of ordinary differential equations (ODE) governing tool dynamics, thus resulting in a mathematical formulation of the Class II milling models.

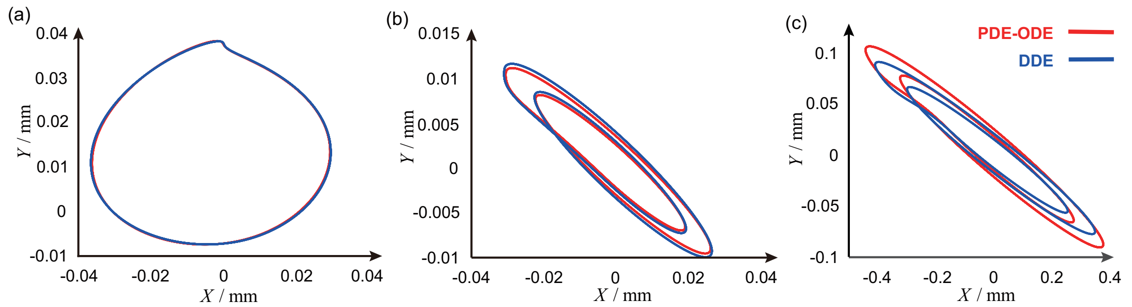

Figure 1: Limit cycles of tool center vibration: (a) radial immersion a_e=100%, depth of cut a_p=1 mm, (b) a_e=10%, a_p=1 mm, and (c) a_e=5%, a_p=20 mm. Spindle speed 30000 rpm, feed rate 0.2 mm/tooth, details of the model parameters can be found in reference [6].

Results and discussion

Case studies have been carried out to compare the proposed PDE-ODE formulation and the classical DDE formulation on specific examples described in [6]. Fig.1 illustrates the limit cycles of tool vibration for three different cases. The two formulations agree with each other when the depth of cut is small. However, the limit cycle predicted by the PDE-ODE formulation is more accurate when the axial depth of cut is large, which is because the simplifications made in the DDE formulation is avoided in the PDE-ODE one. The proposed formulation serves as the mathematical foundation of the Class II models and brings a new perspective for analyzing the milling process.

References

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