Bistability in pressure relief valve dynamics

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Abstract. The vibrations of pressure relief valves hinder the discharging process, thus, this phenomenon must be avoided. The vibrations emerge via Hopf bifurcations, which can be either super- or subcritical. The latter case involves bistable regions and hysteresis effect. The nonlinear analysis is essential for the exploration of the safe parameter zones. Bifurcation analysis is provided in a wide range of parameters comparing analytical, semi-analytical and numerical results.

Introduction

Direct spring operated pressure relief valves protect systems from unsafely high pressure. Despite the crucial safety role, harmful valve vibrations can emerge during operation leading to an insufficient discharge speed and noise. The dimensionless mathematical model of a valve connected to a vessel is already provided in the literature [1, 2] with the state variables y_1, y_2 and y_3 (see Fig. 1 left):

$$\dot{y}_1 = y_2\,, \quad \dot{y}_2 = -2\zeta y_2 - (y_1 + \delta) + y_3\,, \quad \dot{y}_3 = \beta\big(q - y_1\sqrt{y_3}\,\big)\,. \tag{1}$$

Hopf bifurcations were detected with either supercritical or subcritcal characteristics, however, detailed analysis can not be found in the literature in view of the wide range of the parameter combinations. This work aims at revealing the safe parameter domains of system stiffness β, damping ratio ζ and dimensionless inlet flow rate q for fixed value of the set pressure δ. In order to guarantee the safe operation, not only the stability boundaries and the type of the Hopf bifurcations have to be determined but also the bistable solutions are to be indicated.

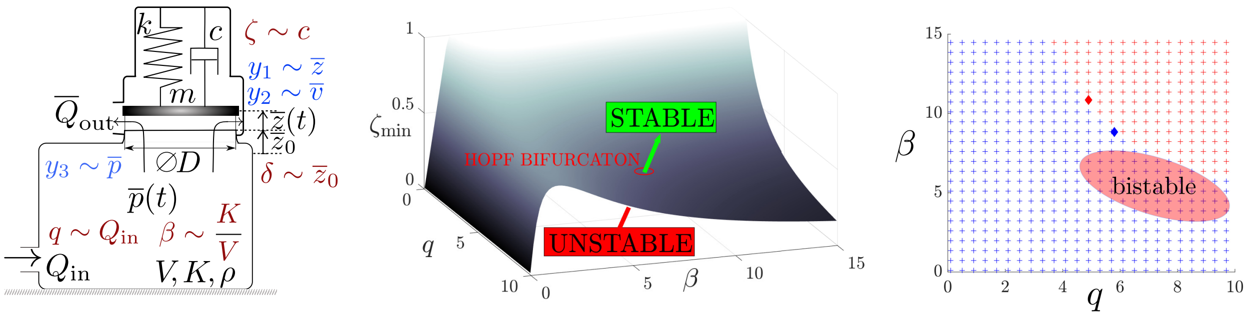

Figure 1: Mechanical model, dimensionless state variables and parameters (left), result of the linear stability analysis (middle), result of the nonlinear analysis (right)

Methods and results

As the result of the linear stability analysis, the Hopf bifurcation points define a surface in the parameter space (q, β, ζ) shown in Fig. 1 in the middle, where the stable parameter combinations are above the surface $(\zeta > \zeta_{\text{min}})$. Operation with $\zeta > \zeta_{\text{min}}$ still cannot prevent vibrations, because in case of the subcritical Hopf bifurcations, an unstable limit cycle surrounds the stable equilibrium restricting the extent of the basin of attraction. This phenomenon leads to bistable solutions that mean coexisting stable equilibrium and stable periodic orbit. In Fig. 1 on the right, a bifurcation type map is shown in the (q, β) plane for the bifurcation points at ζ_{min} by means of DDE Biftool. Blue crosses refer to the supercritical Hopf bifurcations and red crosses show the subcritical ones. The map was checked also analytically with the help of the Hopf bifurcation calculation including centre manifold reduction; this is illustrated by diamonds. Bifurcation diagrams were used to detect fold bifurcations in the limit cycle branches, which can create a globally subcritical characteristic even if the local characteristic is supercritical. The bifurcation diagrams were produced with continuation method, by analytical Hopf calculations, and from simulations by Runge-Kutta method. The location of these possible bistable solutions are also illustrated in the right panel of Fig. 1. In conclusion, the stable domains bounded by supercritical stability boundaries without folds are located in the domain of relatively small values of q and β, moreover the minimal damping ratio can be as high as an external damping is necessary for safe operation.

References

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