Analysis on nonlinear stiffness isolators revealing damping thresholds

Mu-Qing Niu[*] and Li-Qun Chen [*]

[*]*School of Science, Harbin Institute of Technology, Shenzhen, Guangdong, China*

Abstract. An analytical method is proposed to predict all the possible frequency responses of vibration isolators with nonlinear stiffness under different damping. The amplitude-frequency response relation is derived from a harmonic balance method as an algebraic equation. The whole damping region is divided into large damping, medium damping and small damping sub-regions according to the root conditions of the equation. The damping thresholds are obtained, and the frequency response in each sub-region is predicted. Nonlinear phenomena of full-band isolation, bounded response and unbounded response are revealed, and the sufficient and necessary condition for each case is presented. Simulations are performed on nonlinear isolators with cubic stiffness, fifth-order stiffness, and arctangent stiffness, respectively. The damping thresholds and typical frequency responses at each damping sub-region are demonstrated, and the results are verified by the numerical integration based on Runge-Kutta method.

Introduction

For a nonlinear stiffness isolator, the vibration equation can be transformed into an algebraic equation based on the harmonic balance method. In most existing researches, responding amplitudes are solved from the algebraic equation for a given excitation frequency, and the analysis is based on the root condition. The multivalued amplitudes correspond to the jump phenomenon [1]. However, the method is unsuitable for isolators with high-order stiffness due to the difficulty in solving high-order equations, and the analysis is usually restricted to a limited damping region for simplification. The algebraic equation is a quartic equation of the frequency regardless of the stiffness order. It shows promise in proposing a general analysis method for nonlinear stiffness isolators based on the frequency root condition. All the possible frequency responses are expected to be predicted covering the whole damping region. In addition, the multivalued frequencies endow the frequency responses with different features, and new nonlinear phenomena can be exhibited.

Results and discussion

An analytical method is proposed for the isolator with a symmetric nonlinear restoring force. The whole damping region is divided into large damping, medium damping and small damping sub-regions corresponding to single-root, intersecting multi-root and non-intersecting multi-root conditions of the frequency, respectively. The multivalued amplitudes are analysed through the derivatives of the frequency solutions, and the damping thresholds for jump phenomena are obtained. The frequency response in each damping sub-region can be predicted. A single-valued frequency is possible to exhibit a full-band isolation indicating the transmissibility smaller than 1 at all frequencies. Multivalued frequencies at all amplitudes lead to an unbounded response indicating an infinite amplitude. The solvability condition results in a bounded response indicating a limited amplitude regardless of the damping. The sufficient and necessary condition for each phenomenon is presented. The flow chart of the proposed method is shown in Figure 1. Simulations are performed on isolators with cubic stiffness, fifth-order stiffness, and arctangent stiffness, respectively. The damping thresholds of the cubic stiffness isolator are shown in Figure 2, and typical frequency responses in each damping sub-region are demonstrated in Figure 3. A further study incorporated with analytical and numerical methods is performed to reveal the characteristics of the unbounded response.

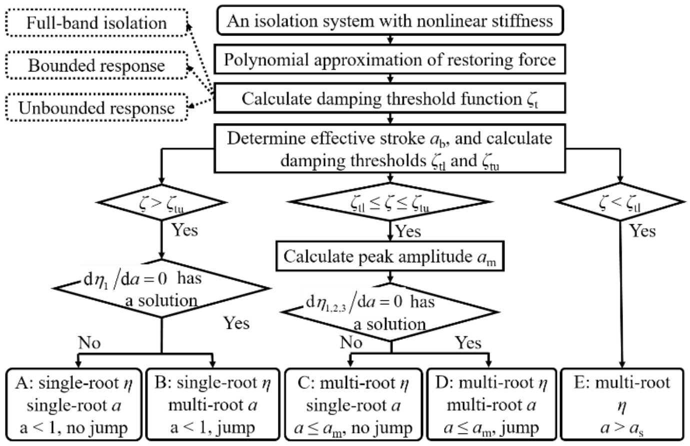

Figure 1: Flow chart of the analysis procedure

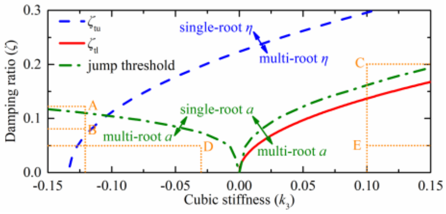

Figure 2: Damping thresholds of cubic stiffness isolator

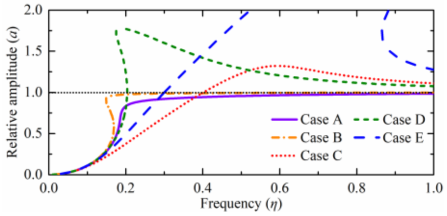

Figure 3: Frequency responses of cubic stiffness isolator

References

[1] Nayfeh A. H., Mook D. T. (1995) Nonlinear Oscillations. Wiley-VCH, Weinheim.