Multiple Periodic Symmetric Limit Cycles of Two Coupled Sommerfeld Rotors

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Abstract. The paper extends the Sommerfeld rotor to two unbalanced, rigidly coupled, elastically supported rotors which rotate in a synchronous operation with the same rotation speed, when both driving moments are equal. For equal moments with opposite signs, there is an asynchronous operation with the same limit cycle in the phase plane of displacement and velocity of the horizontal rotor vibrations. New forms of symmetric limit cycles are calculated when the applied driving moments are unequal. For high-speed rotations, driving moments are applied near the ratios 1:1, 1:3 and 1:4 in order to derive symmetric forms of single, triple or quadruple periodic limit cycles, respectively. For the special case that the rotations of both rotors are rigidly coupled, a new speed moment characteristic is calculated by means of time-free Fourier solutions.

Introduction to Sommerfeld rotors

The Sommerfeld rotor [1] is extended to two unbalanced viscos-elastically supported rotors shown in Figure 1. The application of same driving moments m_1 and m_2 to both rotors leads to vibrations of rotor displacement ξ and velocity ζ described by the one-periodic limit cycle shown in Figure 2. The rotation speeds of both rotors are investigated by means of the mean values $E(\nu)$, $E(\eta)$ and the fluctuating parts $\Delta\nu$, $\Delta\eta$, respectively.

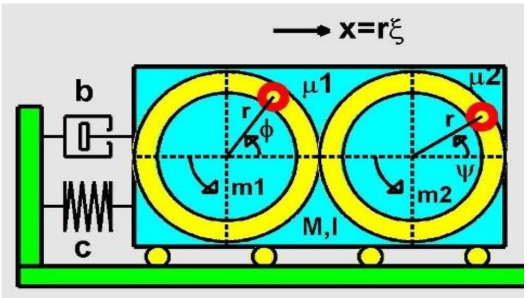

Figure 1: Airplane wing model with two rotors

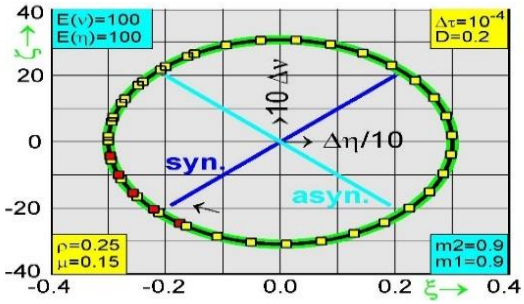

Figure 2: Limit cycle of displacement and speed

New results are derived when both rotations are operated, independently. Figure 3 shows the triple periodic, double symmetric limit cycle when the ratio of the driving moments is near 1:3. In Figure 4, a quadruple limit cycle is calculated where sharp cycle lines are extended to limit flows because $E(\nu)/E(\eta)$ differs from $1/4$.

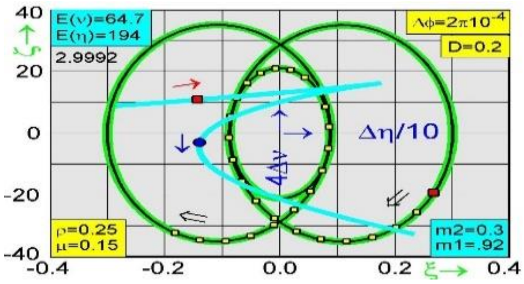

Fig. 3: Triple periodic symmetric limit cycle

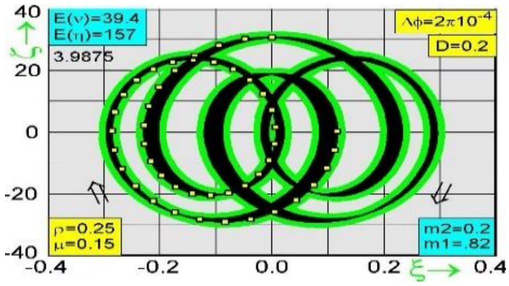

Fig 4: Quadruple periodic limit cycle flow

Limit cycles of time-free rotor equations

The paper introduces orthogonal equations of motion [2] into rotor dynamics where time is substituted by angle [3]. This avoids almost periodic solutions. Stability can be investigated by means of the Floquet theory. Approximated solutions are calculated by means of Fourier expansions. In case that both rotations are coupled, the paper derives a new speed moment characteristic which corrects the classical version by means of the moment of inertia of the unbalanced mass related to the mass moment of inertia of the entire rotor.

References

[1] Sommerfeld A. (1902): Beiträge zum dynamischen Ausbau der Festigkeitslehre, Zeitschrift des Vereins Deutscher Ingenieure, Vol. 46, pp. 391–394 (in German).

[2] Wedig W.V. (2022): Multiple Sommerfeld Effects in Vehicle Dynamics. In Lacorbonara, W., Balachandra, B, Lamey, M.J., Ma,J., Machado, J.A.T. Stepan, G. (eds) Advances in Nonlinear Dynamics, Proceedings of the Second International Nonlinear Dynamics Conference, Vol 2, pp.538-550.

[3] Wedig W.V. (2021): Speed Oscillations of a Vehicle Rolling on a Wavy Road. In Special Issue "Performance and Safety Enhancement Strategies in Vehicle Dynamics and Ground Contact", *Appl. Sci.*,*11*(21). *Sci.*,*11*(21).