Horizontal table vibration for parts-centering without feedback

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Abstract. We study the slow average motions of a particle on a horizontally vibrating frictional table. The equations of motion have non-analytic nonlinearities. Numerical simulations show multiple time scales. The system is analyzed using the method of multiple scales (MMS). The slow-flow integrals are found using asymptotics, have logarithmic nonlinearities, are valid near the target location on the table, are easy to integrate numerically, and retain parametric excitation in slow time. The slow flow matches well with full numerical solutions. This is the first MMS analysis of a problem in this area that we are aware of.

Introduction

A point mass m under gravity g moves on a table with a coefficient of friction μ. The table kinematics is described by its instantaneous centre of rotation $(x_c(t) = R\cos(\Omega t), y_c(t) = R\sin(\Omega t))$ and its angular velocity $\nu = H\cos(\omega t)$. This prescribed motion, for some parameter choices, generates a stable fixed point in space. This problem is of interest in robotics [1, 2] and is relevant to open-loop manipulation of part feeders in industry. We scale length and time by selecting $R = 1$ and $\omega = 1$. Further, letting $H = \epsilon A$, $\Omega = 1/2 + \epsilon\Delta$, and $\mu g = \epsilon^2\alpha$, the equations of motion are:

$$\ddot{x} = \frac{-\epsilon^2\alpha\left\{\dot{x} + \epsilon A\cos(t)\left[y - \sin(\frac{t}{2} + \epsilon\Delta t)\right]\right\}}{\sqrt{\left\{\dot{x} + \epsilon A\cos(t)\left[y - \sin(\frac{t}{2} + \epsilon\Delta t)\right]\right\}^2 + \left\{\dot{y} - \epsilon A\cos(t)\left[x - \cos\left(\frac{t}{2} + \epsilon\Delta t\right)\right]\right\}^2}}, \tag{1}$$

$$\ddot{y} = \frac{-\epsilon^2\alpha\left\{\dot{y} - \epsilon A\cos(t)\left[x - \cos\left(\frac{t}{2} + \epsilon\Delta t\right)\right]\right\}}{\sqrt{\left\{\dot{x} + \epsilon A\cos(t)\left[y - \sin(\frac{t}{2} + \epsilon\Delta t)\right]\right\}^2 + \left\{\dot{y} - \epsilon A\cos(t)\left[x - \cos\left(\frac{t}{2} + \epsilon\Delta t\right)\right]\right\}^2}}. \tag{2}$$

Results and discussion

The method of multiple scales (MMS) works here. The corresponding slow flow equations can be obtained *via* asymptotic approximations for some integrals. These equations are

$$\frac{4\pi A}{\alpha}\zeta'' = \left[-C\eta' + S\zeta' + \zeta'\right]\ln(E) + \left[C\eta' - S\zeta' + \zeta'\right]\ln(F) - 4CA\eta + 4SA\zeta - G\zeta', \tag{3}$$

$$\frac{4\pi A}{\alpha}\eta'' = \left[-C\zeta' - S\eta' + \eta'\right]\ln(E) + \left[C\zeta' + S\eta' + \eta'\right]\ln(F) - 4CA\zeta - 4SA\eta - G\eta'. \tag{4}$$

In Eqs. 3-4, the superscript $'$ denotes a derivative with respect to T_1. Further, $C = \cos(2\Delta T_1)$, $S = \sin(2\Delta T_1)$, $E = \left[\zeta'^2 - \eta'^2\right]S - 2\zeta'\eta' C + \eta'^2 + \zeta'^2$, $F = \left[\eta'^2 - \zeta'^2\right]S + 2\zeta'\eta' C + \eta'^2 + \zeta'^2$, and $G = 10\ln(2) + 4\ln(A) + 4$. In Fig. 1(a), full solutions for Eqs. 1-2 in the $x - y$ plane and for Eqs. 3-4 in the $\zeta - \eta$ plane are superimposed. The match is excellent. In Fig. 1(b), a comparison between y and η is shown. A zoomed-in portion of the same is shown in Fig. 1(c). The similar match for x and ζ is omitted to save space. We emphasize that fast oscillations of x and y are absent from ζ and η, though slow oscillations are retained. Those can be removed with unconventional calculations that we will present in another paper.

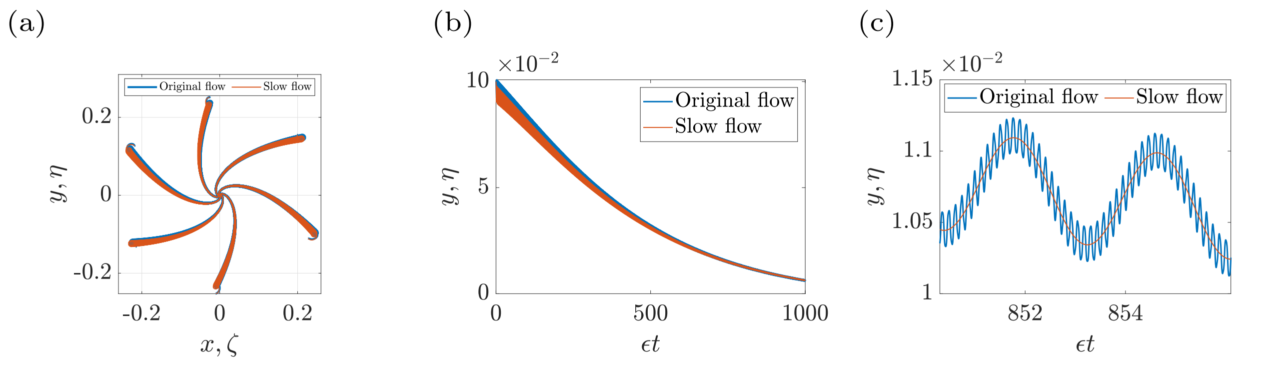

Figure 1: Comparison between solutions of Eqs. 1-2 and Eqs. 3-4: (a) x-y plane, (b) time response for $0 \leq \epsilon t \leq 1000$, and (c) time response for $850 \leq \epsilon t \leq 856$. These results were obtained with parameters $\alpha = 0.5$, $\epsilon = 0.01$, $A = 1$, $\Delta = 1.1$ and for near-zero-velocity initial conditions.

References

[1] Reznik., D. S. (2000) The universal planar manipulator, PhD thesis, University of California, Berkeley.

[2] Vose., T. H., Umbanhowar., P. and Lynch., K. M. (2009) Friction-induced lines of attraction and repulsion for parts sliding on an oscillated plate. *IEEE Transactions on Automation Science and Engineering* **6**:685-699