Dynamic response of spatial beams with material softening and strain localization

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Abstract. Numerical modelling of mechanical response of slender and flexible structures characterized by initiation and propagation of damaged bands involving geometrical and material nonlinearities often results in instabilities. The adopted numerical solution methods become sensitive close to the critical points while investigating such responses. In the present work, we address the problem of strain localization and the dynamic behaviour of the beam-like structural elements at both pre- and post-critical load levels.

Introduction

In the analysis of slender flexible structures undergoing complex deformation, the precise prediction of the mechanical response in the post-critical regime poses a serious challenge for computational methods especially when describing the demanding phenomena such as softening within mathematical constitutive models. The present work focuses on the phenomenon of strain localization in beam-like structural elements which occurs when a material dependent critical condition is reached at some material point of the solid body that results in discontinuities in strain/displacement fields within a thin narrow band. The present work aims at the investigation of dynamic response of the structure undergoing softening at a localized cross-section using the novel energy preserving velocity-based finite element formulation by Zupan and Zupan [1] where typical problems associated with rotational degrees of freedom are completely avoided. The computational advantages of the formulation are preserved after the efficient detection of critical load level and the post-critical treatment of localized strains are implemented into the formulation.

Methodology

The system of governing equations for a three-dimensional Cosserat beam is a set of nonlinear partial differential equations which are as follows [1]:

$$\boldsymbol{n}' + \tilde{\boldsymbol{n}} = \rho A \dot{\boldsymbol{v}}, \tag{1}$$

$$\boldsymbol{M}' + \boldsymbol{K} \times \boldsymbol{M} + (\boldsymbol{\Gamma} - \boldsymbol{\Gamma}_0) \times \boldsymbol{N} + \hat{\boldsymbol{q}}^* \circ \tilde{\boldsymbol{m}} \circ \hat{\boldsymbol{q}} = \boldsymbol{\Omega} \times \boldsymbol{J}_\rho \boldsymbol{\Omega} + \boldsymbol{J}_\rho \dot{\boldsymbol{\Omega}}, \tag{2}$$

where prime($\prime$) denotes the derivative with respect to x and dot $(\cdot)$ denotes the derivative with respect to time, $\tilde{\boldsymbol{n}}$ and $\tilde{\boldsymbol{m}}$ are the external distributed force and moment vectors per unit length, ρ is the mass density and $\boldsymbol{J}_\rho$ is the mass moment of inertia of the cross section, $\boldsymbol{N}$ and $\boldsymbol{M}$ are the vectors of stress resultant force and moment respectively, $\boldsymbol{v}$ and $\boldsymbol{\Omega}$ are the velocities and angular velocities, $\boldsymbol{\Gamma}$ and $\boldsymbol{K}$ are the vectors of translational and rotational strains respectively. In the above equations, the quantities in the fixed basis are denoted in lower case notations and vice versa. The time discretization employed here is based on the midpoint rule while the spatial one is based on Galerkin finite element method. The primary unknowns are chosen to be the velocities and angular velocities due to their numerical advantages of additive-type update procedure and consistency of standard additive-type interpolations when expressed in appropriate reference frame. The proposed methodology performs well and gives accurate results for problems with strain softening.

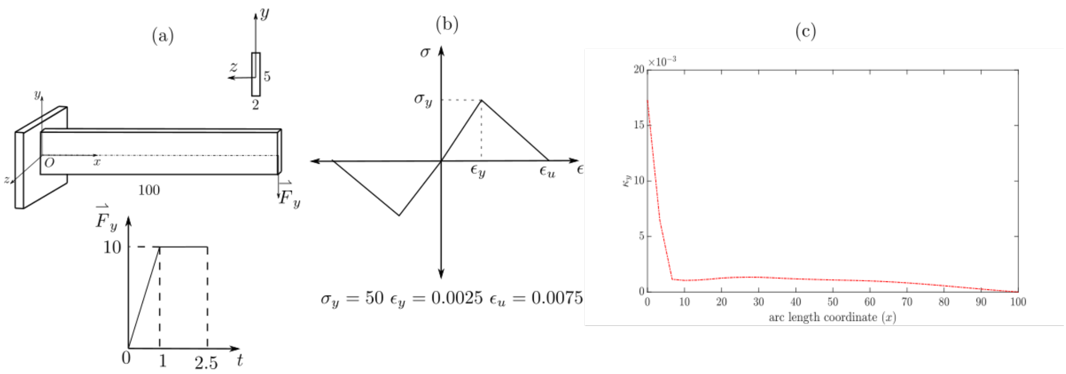

Figure 1: (a) Configuration of the cantilever beam, (b) stress-strain relationship, (c) variation of curvature along the length of the beam.

Figure 1 shows a simple cantilever made of bi-linear material with decreasing stresses after the critical value is reached. In the proposed example, the strains are localized at the clamped end while the obtained bending strain distribution is presented in figure 1(c) at time $t = 2.5$s.

References

[1] E. Zupan and D. Zupan, "On conservation of energy and kinematic compatibility in dynamics of nonlinear velocity-based three-dimensional beams", Nonlinear Dynamics, vol. 95, no. 2, pp. 1379-1394, 2018.