Modal interactions in a non-linear mass-in-mass periodic chain

J. Flosi*, A. Ture Savadkoohi* and C.-H. Lamarque*

**Univ Lyon, ENTPE, Ecole Centrale de Lyon, CNRS, LTDS, UMR5513, 69518 Vaulx-en-Velin, France*

Abstract. The multiple scale dynamics of a forced periodic chain composed of cubic mass-in-mass cells is considered. Dispersion equation of the continuous expression of the system permits modal decomposition of response functions. A particular $1:3$ internal resonance is considered and continuous equations are projected on the two relevant modes. Selection of fast and slow dynamics leads to obtaining the slow invariant manifolds (SIM) and frequency responses of the chain.

Introduction

Firstly introduced in the magnetic field [1], metamaterials are presenting unnatural properties due to their special architecture. Mechanical and vibro-acoustical metamaterials were later designed for control of mechanical energies [2]. Such systems, e.g. mass-in-mass, can be designed to present negative indices. Other variants of this type of metamaterials can be fabricated via multiple inclusions [3] or nonlinearity inclusions [4] leading to multiple tunable bandgaps. In this work, a periodic nonlinear mass-in-mass chain is considered, see Fig.1. In details, it is composed of equally spaced masses m_1 at rest positions which are linearly coupled to each other. Each mass m_1 houses a mass m_2 which possesses a cubic restoring force.

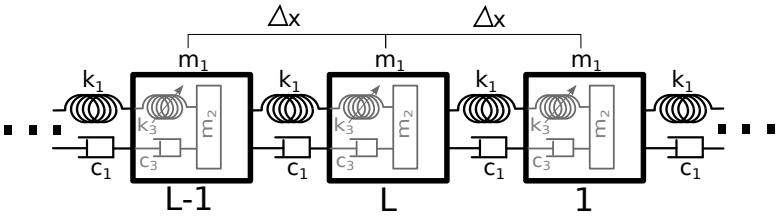

Figure 1: Periodic cubic mass-in-mass chain.

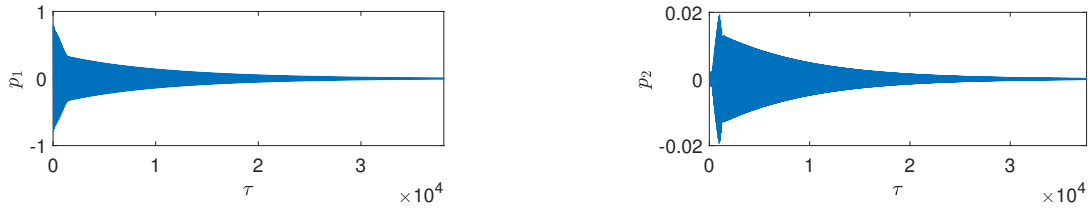

Figure 2: Time histories of modal coordinates of first (p_1) and third (p_2) mode under initial displacement $p_1(0) = 0.75$ and the rest zero

Results and discussion

Inspection of linear part of continuous equations lead to detection of modal characteristics of the system. As an example, a $1:3$ internal resonance is considered. Response functions are decomposed in well chosen bi-modal form and governing equations are projected on two internally resonant modes. Figure 2 presents time histories of modal coordinates of internally resonant modes (first and third mode) represented by p_1 and p_2, respectively. Asymptotic responses [5] of the system are considered. Fast dynamics [6] of the projected system leads to detection of two SIMs associated to two internally resonant modes. Slow dynamics [6] provides singular and equilibrium points, which lead to the prediction of periodic and quasi-periodic behaviors.

References

[1] Veselago V. G. (1968) The electrodynamics of substances with simultaneously negative values of ε and μ. *Soviet Physics Uspekhi* **10**:509-514.

[2] Wu L. et al. (2020) A brief review of dynamic mechanical metamaterials for mechanical energy manipulation. *Materials Today* **44**.

[3] Rodrigues G.S., Weber H.I. (2017) Elastic Metamaterials Analysis: Simple and Double Resonators. *Research Inventy: International Journal of Engineering And Science* **7**:11-16.

[4] Cveticanin L. and Zukovic M. (2017) Negative effective mass in acoustic metamaterial with nonlinear mass-in-mass subsystems. *Communications in Nonlinear Science and Numerical Simulation* **51**:89-104.

[5] Manevitch L. (2001) The Description of Localized Normal Modes in a Chain of Nonlinear Coupled Oscillators Using Complex Variables. *Non Dyn* **25**:95-109.

[6] Ture Savadkoohi A. et al. (2016) Analysis of the 1:1 resonant energy exchanges between coupled oscillators with rheologies. *Nonlinear Dyn* **86**:2145-2159.