Bird like trajectories in 6d chaotic system incorporated with fractional order, memristor and encryption

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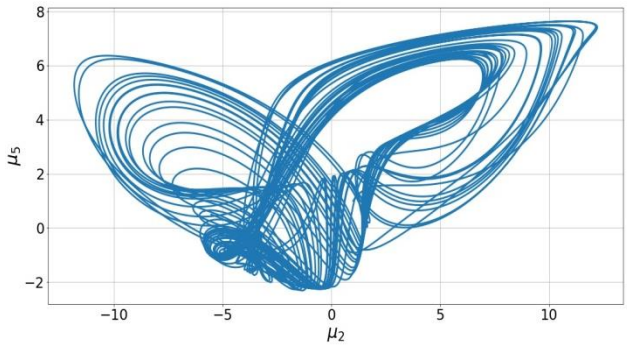
Abstract: This paper reports a novel six dimensional nonlinear dynamical system with the tangent hyperbolic memristor circuit and amalgamated image encryption. The dynamical system is analyzed using standard tools including phase portraits, equilibrium points, Eigenvalues, and Lyapunov exponents. Analysis suggests that the developed system is chaotic in nature and has exiting new 2D trajectories. The system generates two different trajectories of bird like phase portrait. The chaotic system is numerically solved with Caputo fractional order derivative for different values of fractional order (q). The fractional order circuits are designed for q = 1 and q = 0.99 utilizing the approximation fractional order technique of transfer function, showing great deal of agreement with the numerical results. Lastly, the random number generated from the chaotic system is utilized to scramble the image via the scheme of Amalgamated Image Encryption. The scrambled image is tested using different image security test algorithm to support the idea that the chaotic system and image can together form an advantageous key.

Introduction

The nonlinear dynamics of fractional order system and memristor based circuits have attracted an escalating attention in recent years. Memristor chaotic systems (MCS) have been discussed frequently as a new type of chaotic system. Memristor is the abbreviation of memory resistance, a basic component of a circuit system described by Chua in 1971 [1]. During operation, the memristor's resistance changes with the current flowing through the circuit, but it remains unchanged when switched off. A memristor plays a nonlinear role in voltage-ampere characteristics, allowing chaotic systems to perform better. As a result, chaotic systems have widely adopted it, resulting in MCS. Calculus of fractional order derivatives is now becoming an important aspect in the application of engineering and technology applications [2]. The field deals with integer and non-integer arbitrary order values for derivatives that are then applied to the study of various complex physical systems present in the world. The mathematics of fractional calculus requires great computing skills, as the subject is pretty much dependent on complex numerical solutions, complicated simulations and statistical analysis tools. The advantages of the fractional derivative model over the integer derivative model are its robust results, complex nature solution, its memory effects, and heredity assets for different systems.

This study aims to make development in the field of memristor-based chaotic systems. The main idea is to investigate a novel 6D chaotic system with memristor-based system. Due to the unique nonlinear characteristics of the memristor different pseudo-random numbers are generated which are utilized to do cryptography. A new encryption method is presented, “Amalgamated Image Encryption” in which the chaotic data along with its parameter and initial condition, and a plain image is represented as a key for multimedia (i.e., image and video) [3]. The 6D chaotic equation and one of the phase portraits of the novel system is illustrated below.

$$\dot{\mu}_1 = a(\mu_2 - \mu_1) + \mu_2\mu_3 - h\mu_5\mu_4 - k(\alpha + \beta \tanh(\mu_6))\mu_4$$
$$\dot{\mu}_2 = c\mu_2 - \mu_1\mu_3$$
$$\dot{\mu}_3 = -b\mu_3 + \mu_1\mu_2$$
$$\dot{\mu}_4 = \mu_2\mu_3 - c\mu_5 + \mu_3 - \mu_4^3$$
$$\dot{\mu}_5 = \mu_2\,\mu_1 + d\mu_4 - \mu_5 - 0.09\exp(\mu_5)$$
$$\dot{\mu}_6 = \dot{\mu}_4$$



Where, a = 19, b = 3, c =10, d = 12, k = 1; α = 1; β = 0.5 and h = 3

References

[1] Chua, L.J.I.T.o.c.t., *Memristor-the missing circuit element.* 1971. **18**(5): p. 507-519.
[2] Hilfer, R., *Applications of fractional calculus in physics*. 2000: World scientific.
[3] https://doi.org/10.5281/zenodo.7046557