Recent Developments in Bifurcation and Continuation Theory Methodology Applied to Aircraft Flight Dynamics

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Bifurcation Analysis and Continuation Theory Methodology (BACTM) proposed in early 1980s by Raman and Mehra [1] started a new paradigm for many researchers delving into the field of nonlinear dynamics of aircraft. Works of Zagaynov and Goman [2] and Guicheteau [3] helped in understanding flight dynamics of six degrees of freedom models of aircraft stretching operations into nonlinear regimes of flight. Several research works extending from investigation of nonlinear aircraft flight dynamics to using BACTM as aid in aircraft conceptual design and further in control design for fully developed aircraft followed in 1980s and early 1990s. The methodology is based on treating equilibrium and stability of nonlinear dynamical systems characterised by a set of ordinary differential equations,

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{U}), \qquad (1)$$

where $\boldsymbol{x} \in \mathbb{R}^n$ are n-states and $\boldsymbol{U} \in \mathbb{R}^m$ are m-controls of the system. $\boldsymbol{f}$ is nonlinear, atleast piecewise smooth vector field governing the dynamics of the system. Dynamics of the above system can be studied via computation of one-parameter equilibrium curves, stability and bifurcations for which several numerical tools, for example AUTO continuation algorithm, are available in public domain [4]. For different types of one-parameter bifurcations resulting into different parametric time history of the system one may refer to an excellent and one of most popular textbooks written by Strogatz [5]. The practise of using one-parameter continuation for investigating global dynamics of nonlinear dynamical system of the form in Eq. (1) turned out to be restrictive in a sense that it did not work for investigating actual flights of aircraft, which were state or otherwise constrained and required deployment of more than one parameter. Partly, this problem was solved by Lowenberg [6] by continuing with 1-parameter numerical continuation but of the augmented system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{U}); \boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{0} \qquad (2)$$

where $\boldsymbol{g}(\boldsymbol{x})$ represent constraint functions. The computed solutions using this approach were equilibrium states satisfying constraints, however, stability of the constrained equilibrium states turned out to be incorrect and were thereby separately computed and further supplemented with numerical time simulation results to infer the dynamics. All of the above problems of computing stability and bifurcations in constrained conditions were overcome with the invention of the Extended Bifurcation Analysis (EBA) Technique by Sinha and Ananthkrishnan [7] in the year 2000. Some of the key results spanning computation of stability and bifurcation in constrained flight conditions, design parameters, control schedules, flight envelope, etc., based on the EBA technique are presented in Fig. 1 (a,b). Last review on the use of bifurcation and continuation methods to flight dynamics, including the works using the EBA technique, was reported in 2007 in Ref. [8]. Further integration of continuation algorithm with MATLAB via MATCONT helped in automating EBA process using MATLAB subroutines (COSY [9]) for specific problems, for example, related to constrained states and stability constrained control design [10].

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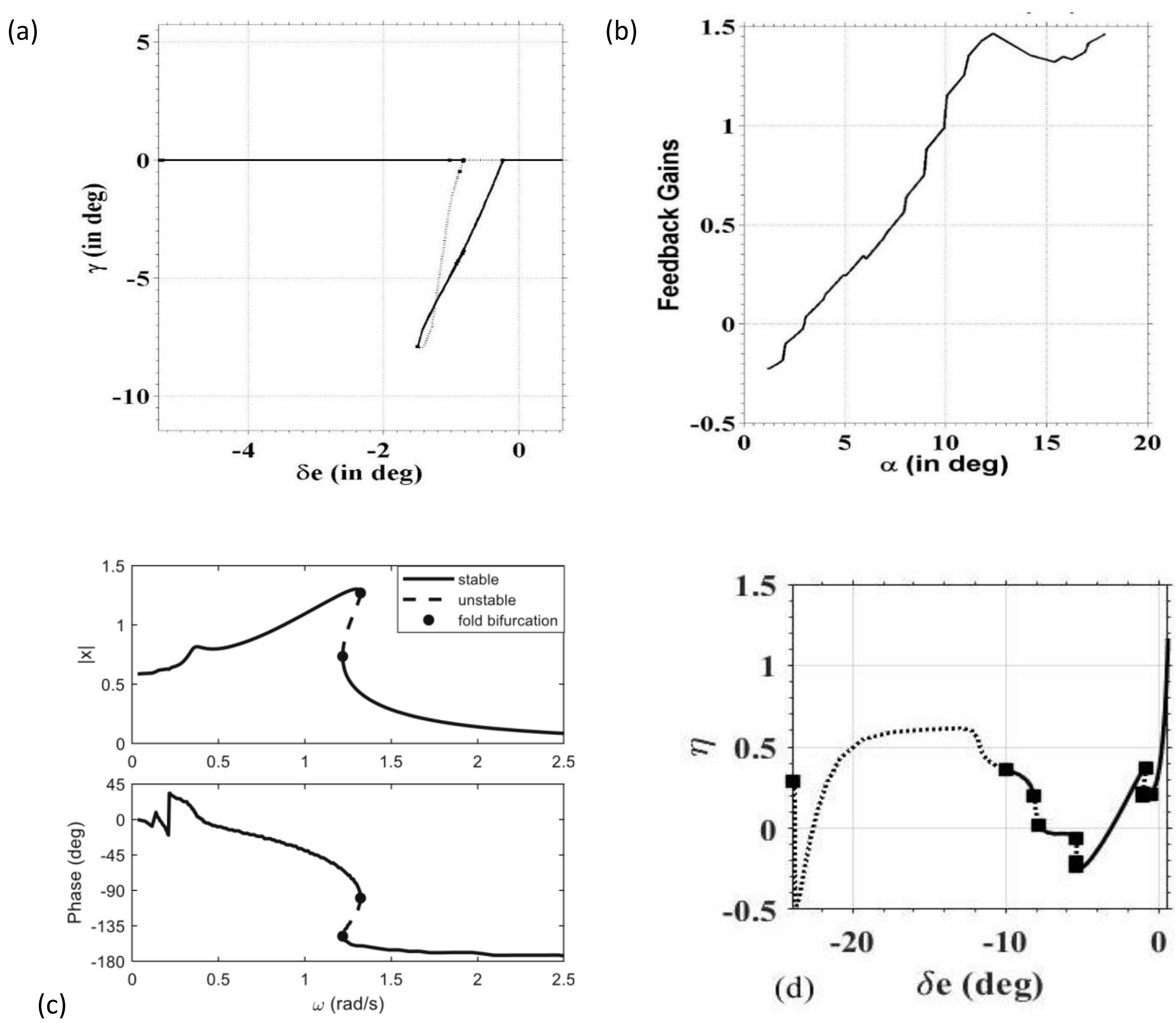

Figure 1: (a) Constrained level flight trims for F-18-HARV model, (b) feedback control gains for fixed short-period response complete range of level flight trims (Ref. [10]), (c) nonlinear frequency response (Ref. [11]) and (d) open-loop control interconnect schedule (Ref. [12]).

Besides using bifurcation results as aid to control law design, most recent works highlighting direct use of continuation algorithm as aid in flight controller performance evaluation for nonlinear aircraft models have been reported by Lowenberg and co-workers in Ref. [11] and design of nonlinear control schedules for spin recovery of aircraft have been carried out by Rohith and Sinha [12]. Some results from Refs. [11,12] are shown in Fig. 1 (c, d).

Figure 1(a) shows continuation results of state constrained (in this case level flight equilibrium solutions characterised by flight path angle $\gamma = 0$) equilibrium solutions of a model of F-18 High-Angle-of-Attack Research Vehicle. Loss of stability and bifurcations from constrained flight conditions are some of the useful outcomes of such computations. This example is an application of direct constraints on aircraft model such as in Eq. (2). Figure 1(b) is an example of implicit constraints on the eigenvalues of the computed level flight trims in Fig. 1(a). Gains are computed in a continuation such that the eigenvalues of a particular mode (in this case short-period) for all the level flight trims are retained at a fixed location. Unique feature of a continuation algorithm allows dealing with a completely nonlinear model of a system coupled with an oscillator, thus facilitating computation of nonlinear frequency response of a system (such as one in Fig. 1(c)) at an equilibrium state of interest. Response of an aircraft to pilot induced oscillation at a flying condition could be thus easily studied via this approach without resorting to the long route of linearizing a model and computing Bode plots. Nonlinear motions of aircraft at very high angles-of-attack, such as spin, are fatal often leading to spatial disorientation and consequent loss of aircraft. Optimal recovery control schedule design from such motions thereby is a very important and challenging task. Open loop control schedules (such as one in Fig. 1(d)) for recovery from spin conditions can be easily obtained using a continuation. These open loop control schedules can be very useful for flight controller designers.

The full article intends to summarize details of works carried out using continuation algorithm with applications to aircraft flight dynamics and control since the last review in 2007. Perspective on future works will also be presented in the full submission.

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