# Accurate asymptotic description of nonlinear friction states for a detailed FEM model

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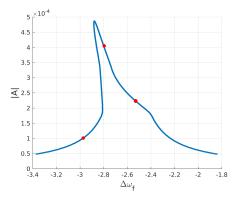
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**Abstract**. This work presents a novel asymptotic method that drastically reduces the computational effort to determine the dynamical behaviour of cantilever slender structures with nonlinear friction at the attachment. The resulting resonance curve for these structures, despite of being frequently computed using a high fidelity finite element model, looks very similar to that from a one dimensional nonlinear oscillator. Therefore, using asymptotic techniques we derive an equivalent one dimensional nonlinear model that captures the physics of the problem in the time scale of interest. The computation of the nonlinear resonance curve with this asymptotic model is reduced to the evaluation of a simple analytical expression, and it shows a very good agreement with full nonlinear FEM simulations.

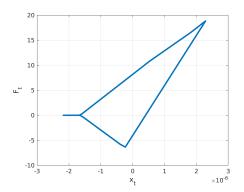
#### Introduction

The accurate computation of the dynamical response of a bladed-disk system usually requires finite element models with millions of degrees of freedom. Each sector presents nonlinear friction effects at the contact interfaces, which are crucial to correctly determine the response of the system but only affect a very small subset of degrees of freedom [1]. These effects produce the dissipation that saturates the vibration amplitude of the system produced by an instability (flutter) or by external excitation close to a resonant frequency (forced response). Since nonlinear friction is small, the system is numerically stiff, and the integration over a large number of elastic cycles is required for the system to converge. However, even when using a highly detailed FEM model, the resonance curve for a cantilever blade with nonlinear friction effects at its root looks similar to that of a one dimensional nonlinear oscillator.

The most commonly used reduction techniques rely on the restriction of the motion of the structure to a finite set of modes (e.g., Craig-Bampton [2]). Nevertheless, the approach in this work is quite different, and it is based in asymptotic perturbative techniques that have already been successfully applied to mass-spring models [3]. Using the fact that nonlinear friction is a small effect, we separate the system into two time scales: a fast one, which corresponds to the oscillations with the elastic frequency of the blade; and a slow modulation, where the effect of nonlinear friction at the contact interfaces is relevant. This method is suited for slender structures with nonlinear friction effects at the attachment, like cantilever blades in a turbomachinery bladed-disk.



(a) Resonance curve computed using the asymptotic model. The red dots are full order model solutions.



(b) Friction cycle of one of the contacts with stick-slip transitions and gap-contact states.

## **Results and discussion**

The complete FEM equations of a detailed structure is reduced to an equivalent one dimensional nonlinear equation. In particular, we apply these techniques to the forced response of a simplified blade like model with a stick-slip friction law and contact-gap configurations (see Fig. 1b). The resonance curve computed from the asymptotically reduced one dimensional equation is shown in Fig. 1a, which just requires the evaluation of an analytical expression. The model is able to capture highly nonlinear effects and shows a very good agreement with the full model, which requires several hours to converge each point in the resonance curve in Fig. 1a.

## References

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