Analysis of forced vibrations of the nonlinear elastic plate on a viscoelastic foundation subjected to hard excitation from harmonic load

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Abstract. In the present paper, the dynamic response of a nonlinear Kirchhoff-Love plate resting on a viscoelastic foundation in a viscoelastic medium, damping features of which are described by the Kelvin-Voigt fractional derivative model is studied for the case of forced vibrations excited by harmonic load. The damping properties of the viscoelastic Fuss-Winkler-type foundation are described by the fractional de-rivative standard linear solid model. Supposing that only two natural modes of vibrations strongly coupled by the internal resonance 1:1 are excited, the generalized method of multiple time scales in conjunction with the expansion of the fractional derivative in terms of a small parameter has been utilized for solving nonlinear governing equations of motion. Assuming the amplitude of external load to be hard, resolving equations are obtained for determining nonlinear amplitudes and phases for various types of external resonances that may occur in systems with cubic nonlinearities.

Introduction

The problem of vibrations of a plate on a viscoelastic foundation has many engineering applications, and therefore is of great interest to many researchers. In order to describe the viscoelastic behavior of Fuss-Winkler-type or Pasternak-type foundations, different models of viscoelasticity have been suggested during the last few decades [1,2]. Apart from the well-known models for viscoelastic materials, such as Kelvin–Voigt, Maxwell or standard linear solid models, one may refer to their fractional derivative generalizations [3], which are becoming increasingly widespread nowadays.

Previous analysis of the free damped vibrations of an nonlinear elastic plate on a viscoelastic foundation has shown [4], that the one-to-one internal resonance could occur in such a mechanical system when the natural frequencies of two coupled modes are close to each other. This problem was generalized for the case or force-driven vibrations of the plate subjected to moving harmonic load in conditions of combination of internal and external resonances [5].

Professor Naife in his monographs [6,7] showed that when analyzing the response to an external harmonic load, four cases should be distinguished depending on whether the excitation is “soft” or “hard”, resonant or nonresonant [6,7]. It was also demonstrated that in addition to primary resonance in systems with cubic nonlinearities secondary resonances could occur under the hard excitation, namely, superharmonic resonance and subharmonic resonance [6].

Results and Discussion

The main purpose of the present research is to study the effect of hard excitation from harmonic load on the forced nonlinear vibrations of a von Karman elastic plate resting on a viscoelastic Fuss-Winkler-type foundation, the damping features of which are described by the fractional derivative standard linear solid model. The generalized method of multiple time scales in conjunction with the expansion of the fractional derivative in terms of a small parameter has been utilized for solving nonlinear equations of motion for the case when amplitude of applied force is large value. The governing equations are obtained and solved numerically for determining of nonlinear amplitudes and phases for the cases when internal resonance one-to-one is accompanied by the superharmonic or subharmonic resonances excited in nonlinear plate. The effect of nonresonant hard excitation on the amplitude and frequency of undamped and damped vibrations of the plate is shown, which depends on the amplitude of applied force. The resulting sets of equations for all types of excited resonances allows one to control also the damping properties of the surrounding medium and the foundation by changing the fractional parameters from zero to unit.

References

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