Harmonic expansion and nonsmooth dynamics in a circular contact region with combined slip-spin motion

Mate Antali

Department of Applied Mechanics, Szechenyi Istvan University, Gyor, Hungary

Abstract. We analyse a rigid body in planar motion while touching a rough plane at a finite-sized, circular contact area. By assuming Coulomb friction between the tangential and normal pressure distributions, the resultant forces and torques can be expressed formally with a nonsmooth dependence on the kinematic variables. In the literature, the exact form of the tangential forces is available for special pressure distributions expressed by transcendent functions; recently, an approximate linear model was introduced. Now, we present a nonlinear extension of the approximation, which can be used effectively to characterise slipping-sticking transitions between the bodies.

Introduction

Consider a rigid body touching a fixed, rough plane in a finite-sized circular contact area (see Fig. 1). Rigid body motion is assumed in the plane xy of the contact region. This motion is parametrized by the velocities u_x, u_y of the contact point and the angular velocity ω_z. The normal pressure distribution is assumed to be constant in time and to have the circular symmetric form $p(x,y) = \tilde{p}(\sqrt{x^2+y^2})$. Consider permanent slipping, thus, $u_x = u_y = \omega_z = 0$ is excluded.

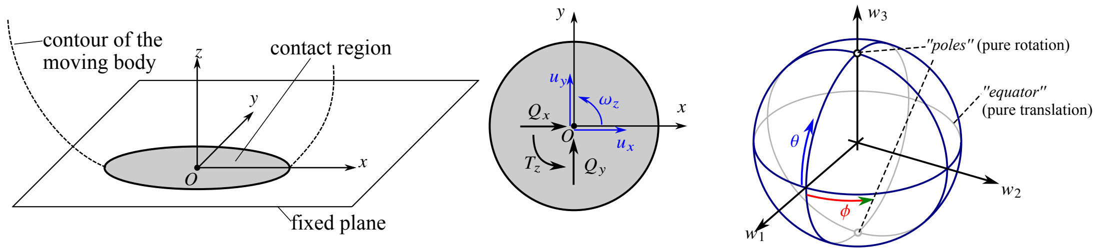

Figure 1: Left and middle panel: Kinematic and dynamic variables in the tangent plane of the circular contact area. Right panel: parametrizing the direction of slipping by the variable sets (w_1, w_2, w_3) and (θ, ϕ).

Results and conclusion

According to [1], let us introduce the variables $(w_1, w_2, w_3) = (u_x, u_y, \rho\omega_z)/\sqrt{u_x^2+u_y^2+\rho^2\omega_z^2}$ to express the *direction* of the relative motion, where ρ is a scaling parameter. The variable set (w_1, w_2, w_3) lies in the unit sphere $\mathbb{S}^3$ as $w_1^2+w_2^2+w_3^2 = 1$. This unit sphere can also be parametrised by two angles θ and ϕ in the form $w_1 = \cos\theta\cos,\phi$, $w_2 = \cos\theta\sin\phi$, $w_3 = \sin\theta$. Assume Coulomb friction model with a friction coefficient μ between the pressure distributions. Then, the resultant force system contains tangential forces Q_x, Q_y and the normal torque T_z (see Fig. 1). According to CITEE, the circular symmetric case leads to the form

$$
\begin{aligned}
Q_x &= -\mu P \frac{w_1}{\sqrt{w_1^2+w_2^2}} \cdot \mathcal{Q}_w(w_3) = -\mu P \cos\phi \cdot \mathcal{Q}(\theta), \\
Q_y &= -\mu P \frac{w_2}{\sqrt{w_1^2+w_2^2}} \cdot \mathcal{Q}_w(w_3) = -\mu P \sin\phi \cdot \mathcal{Q}(\theta), \\
T_z &= -\mu P \lambda \cdot \mathcal{T}_w(w_3) = -\mu P \pi_{xy} \cdot \mathcal{T}(\theta),
\end{aligned}
$$

where the normal force P and the coefficient λ are computed from $\tilde{p}$. The algebraic form of the functions $\mathcal{Q}_w, \mathcal{T}_w, \mathcal{Q}, \mathcal{T}$ can be computed for constant [3] and parabolic [2] pressure distributions. In [1], the authors use the approximation $\mathcal{Q}_w \approx \sqrt{1-w_3^2}$ and $\mathcal{T}_w = w_3$, which makes the above tangential forces and moment linear in w_3 and *harmonic* in θ, which lead to qualitative analysis of the nonsmooth dynamics. However, by this approximation, some bifurcations can be lost.

Now, we will expand the expressions by the truncated Fourier expansion of $\mathcal{Q}$ and $\mathcal{T}$. We show that then, nonlinear terms appear in $\mathcal{Q}_w, \mathcal{T}_w$, and we can extend the nonsmooth dynamical analysis of [1] to find the bifurcations of fixed points on the sphere of $w_1^2+w_2^2+w_3^2 = 1$. The fixed points of this fast subsystem correspond to the limit directions where transitions are possible between slipping and sticking of the body. Our goal is to determine the combined slip-spin states just before sticking.

References

[1] Antali M., Varkonyi P. L. (2022) The nonsmooth dynamics of combined slip and spin motion under dry friction. *Journal of Nonlinear Science* **32**(4) paper 58.

[2] Leine R.I., Glocker C. (2003) A set-valued force law for spatial Coulomb-Contensou friction. *Eur. J. Mech. A* **22**(2): 193–216.

[3] Magyari E., Weidman P.: (2020) Frictionally coupled sliding and spinning on an inclined plane. *Physica D: Nonlinear Phenomena* **413**: paper 132647.