Anti-synchronization of Quaternion Valued Inertial Neural Networks with Unbounded Time Delays: non-reduction, non-separation method

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Abstract. In this article, Global anti-synchronization for a class of quaternion-valued inertial neural networks (QVINNs) with unbounded delays is considered. Based on the two different types of control strategies (feedback and adaptive controllers) and Lyapunov stability theory, various fruitful criteria are obtained to ensure the global anti-synchronization of QVINNs. Most of the synchronization and anti-synchronization results for QVINNs are based on the variable substitution approach, which reduces the order of the original second-order system into the first-order system; also, quaternion-valued networks are separated into four equivalent real-valued neural networks which are made the analysis process more complicated. However, in the present study, the authors deal with the non-reduction order method and non-separation approach for QVINNs, making the analysis approach more concise and easier to deal with second-order neural networks. Finally, two numerical example are provided to shows the effectiveness of our proposed model

Introduction

Quaternion was first introduced by Hamilton (1853) [7], which is a noncommutative division algebra. Because of the non-commutativity, quaternion research is much more difficult rather than real-valued neural networks (RVNNs) and complex-valued neural networks (CVNNs). This is the major reason for the slow research of the quaternion-valued neural networks (QVNNS). In order to avoid the non-commutativity, some researchers separated the QVNNs into four equivalent RVNNs or two CVNNs to study these problems. However, the separation method increases the dimension of the original systems, which leads to difficulties for mathematical analysis [3]. Luckily, as modern mathematics has advanced and expanded, applications of quaternion for future development have been discovered in recent years. It has good application prospects in three-dimensional and four-dimensional data modeling; also, quaternion has gained increasing attention in various fields, for example, attitude control, computer graphics, etc., [2] etc.

In the last few years, anti-synchronization results have been widely used in many areas, including communication processing and information systems [1]. When using anti-synchronization in communication systems, we can transmit digital signals continuously between anti-synchronization and synchronization to strengthen secrecy. As a result, synchronization or anti-synchronization analysis of nonlinear systems has become more popular. Unfortunately, up to now, the study on exponential anti-synchronization of QVNNs with unbounded delays and a non-separation approach has not been involved.

This article will give multiple new results about the anti-synchronization of QVNNs as follows:

1. This article dealt with the global exponential anti-synchronization of QVNNs with unbounded time-varying delays.
2. This article dealt with the QVNNs with inertial terms by utilizing the non-reduction order and non-separation approach.
3. in this article, authors first time designed the adaptive controller for QVNNs, which are used to study the non-reduction approach for inertial terms in QVNNs.

Results and discussion

Unlike the traditional variable substitution approach for the reduced order method, the article's authors dealt with the global anti-synchronization of QVNNs with unbounded time-varying delays and inertial terms. By constructing the Lyapunov functional and designing a linear quaternion-valued feedback and adaptive controllers, some innovative conditions with less conservative results are obtained in Theorem [1,2] to ensure the global anti-synchronization for QVNNs with the non-separation method. The results of this article are more compact and less calculative due to the non-separation approach [3,4]. Since the time-varying delays are unbounded in this article, the proposed QVNNs are more general and execute wider applicability.

The method adopted in this article is totally new, which motivates the researchers to further study of QVNNs with non-separation and non-reduction methods for finding the fixed time and pre-assigned fixed time stability.

References

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