Dynamics of Delayed Piecewise Linear Mathieu Equation

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Abstract. We study the dynamics of a piecewise linear (PWL) oscillator subjected to parametric excitation, time delayed feedback and cubic nonlinear interaction force. The governing equation is non-smooth, essentially nonlinear and infinite-dimensional. In the absence of cubic nonlinearity, the solutions are scalable, i.e., if $\phi(t)$ is a solution, then $\alpha\phi(t)$ $(\alpha \neq 0)$ is also a solution. We render the dynamical system finite dimensional by using Galerkin approximation and evaluate the Lyapunov-like exponent to explore the regions of stability. The method of averaging (MAV) is invoked to derive slow-flow equations to explore the stability of periodic solutions and bifurcations thereof.

Introduction

PWL oscillators are isochronous in spite of their essential nonlinearity. A cracked beam [1] exhibits PWL behaviour wherein the effective stiffness is higher during crack closure phase in comparison to that of crack opening. Systems which have intermittent contact and/or backlash [2] are effectively modeled as PWL systems. As such, analytical study of their dynamical behavior is of interest and importance. To this end, we consider a PWL Mathieu equation [2] [3] with cubic nonlinearity and time delay in the following non-dimensional form

$$\ddot{u} + \Big\{\kappa(u) + \varepsilon\, P \sin(\Omega t)\Big\}u + \varepsilon\, C\, u^3 = \varepsilon\, D\, u\,(t-1)\,. \tag{1}$$

Where time delay has been rescaled to unity, P, C, and D scaled by $\varepsilon(0 \leq \varepsilon << 1)$ are the amplitude of the parametric excitation, strength of cubic nonlinearity, and delayed feedback respectively, Ω is the frequency of the parametric excitation, $\kappa(u) = k_1^2$ for $u > 0$ and $\kappa(u) = k_2^2$ for $u \leq 0$. The time period and natural frequency of the unperturbed autonomous oscillator ($\varepsilon = 0$) is $T = \pi(1/k_1 + 1/k_2), \omega_{pwl} = 2\pi/T$ respectively. The excitation frequency is considered close to a resonance manifold such that $\Omega = m\omega_{pwl} + \varepsilon\sigma$, where $m \in \mathbb{Z}^+$ and $\sigma = O(1)$ is the frequency detuning parameter.

Results and discussion

We begin with the linear system ($C = 0$) and use Galerkin approximation to render the infinite-dimensional dynamical system a finite-dimensional one. Owing to the scalability of the system, we evaluate the Lyapunov-like exponents [3] to explore the stable and unstable regions in the $\sigma - P$ plane (Fig. 1(a)). MAV is invoked by considering PWL basis functions [4] for the unperturbed autonomous system and derive the slow-flow equations. The fixed points of the slow-flow equations correspond to the steady state solutions of Eq. (1) and forms the boundary in Fig. 1(a) (green curve). In case of a nonlinear system ($C \neq 0$), there are multiple steady state solutions and the bifurcation plot is shown in Fig. 1(b). The steady state solutions undergo saddle-node bifurcation at $\sigma = \sigma_1$, supercritical pitchfork bifurcation at $\sigma = \sigma_2$, subcritical pitchfork bifurcation at $\sigma = \sigma_3$ and saddle-node bifurcation at $\sigma = \sigma_4$. From the MAV, we observe that there exists no trivial steady-state solutions before $\sigma = \sigma_1$ and after $\sigma = \sigma_4$ in Fig. 1(b). Fig. 1(c) shows the three-dimensional bifurcation plot in $\sigma - k_2 - A_a^*$ space. By decreasing the value k_2 to 1.5174, we eliminate subcritical pitchfork bifurcation. Below $k_2 = 1.5174$, we have only two saddle-node bifurcation of the trivial solution $A_a^* = 0$.

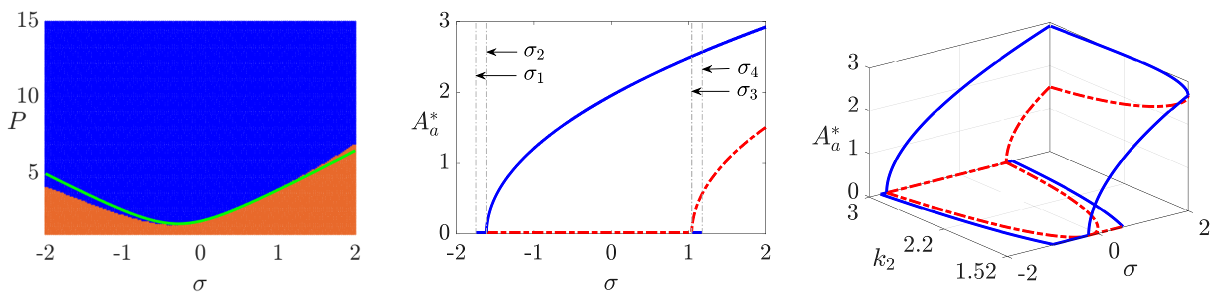

Figure 1: (a) Stability chart, orange: stable, blue: unstable and green: steady state solution of slow-flow equation ($C = 0$). (b) Bifurcation diagram for $k_2 = 3$. (c) Bifurcation diagram in $\sigma - k_2 - A_a^*$ space, (dotted lines: unstable, and solid lines: stable solutions) corresponding to $k_1 = 1$, $\varepsilon = 0.1$, $P = 4$, $C = D = 1$, $m = 1$.

References

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