Nonlinear modeling of micro-cantilever beams

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Abstract. In this work, we analytically examine the validity of using a standard nonlinear beam model of an electrostatic micro-cantilever beam compared to a geometrically exact theory that incorporates shear deformability, nonlinear bending curvature, inertia and longitudinal inextensibility. The results show that the geometrically exact model is more suitable to accurately capture the nonlinear behavior of electrostatic MEMS designs.

Introduction and Problem Formulation

The nonlinear characteristics of electrostatically excited micro-cantilever beams play an important role in determining their performance and suitability to design and manufacture useful devices. Here we provide analytical evidence of the importance of a geometrically exact approach when modeling electrostatic cantilever microbeams. The governing equation of motion of the microbeam is a consistent expansion of the geometrically exact equation of motion obtained in [1] and reads:

$$(1-\frac{1}{2}\hat{v}'^2)\ddot{\hat{v}} + \hat{v}' \int_0^{\hat{s}} (\hat{v}'\ddot{\hat{v}}' + \dot{\hat{v}}'^2) d\hat{\xi} + \hat{c}\dot{\hat{v}} + \hat{v}'' \int_{\hat{s}}^1 \Big[\hat{v}'\ddot{\hat{v}} - \int_0^{\hat{\xi}} (\hat{v}'\ddot{\hat{v}}' + (\dot{\hat{v}}')^2) d\hat{z}\Big] d\hat{\xi} + \Big[\hat{v}'''' + \hat{v}''(\frac{1}{2}\hat{v}''\hat{v}'^2) - \hat{v}'' \int_{\hat{s}}^1 \hat{v}''\hat{v}''' d\hat{\xi}\Big] = \beta\, f_{es}$$

where v denotes the transverse motion, the prime and over-dot indicate differentiation with respect to the arc-length and time, respectively, and f_{es} indicates the electrostatic force per unit length.

Results and Discussions

Variation of the tip-section static equilibria with the DC voltage was obtained by eliminating the time derivatives in the equation of motion and employing a ROM with one-, two-, and three-mode projection in the Galerkin expansion. We note that an odd number of mode shapes leads to faster and closer convergence than using an even number, see Fig. 1(a). Henceforth, we adopt the odd-mode ROM approximation in the rest of the static analysis. Then, we compare the results to those obtained using a standard nonlinear beam model [2] with three-modes projection and 2D-FEM models as shown in Fig. 1(b). A good agreement is observed between the proposed model and FEM results, however, the pull-in voltage is slightly higher than FEM model. This, in fact, is expected since the model is stiffer and requires more voltage to pull it down.

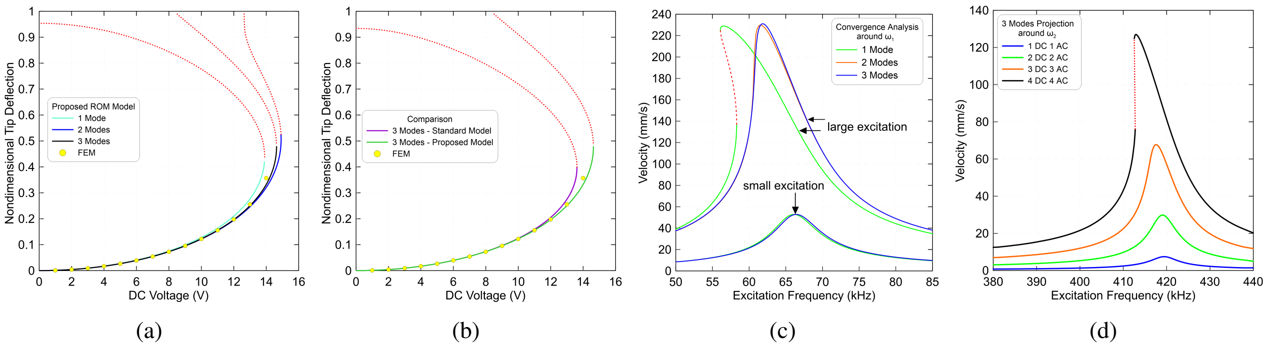

Figure 1: The tip deflection vs. DC voltage obtained considering: (a) 1-, 2- and 3-mode projection and (b) comparison of the standard and geometrically exact models. (c) Dynamic convergence analysis using the proposed model with 1-, 2- and 3-mode projections in the vicinity of the first natural frequency. (d) FRCs around the second natural frequency.

We carried out a dynamic convergence analysis to determine the minimum number of modes required in the expansion. To this end, we obtained the frequency response curves in the vicinity of the first natural frequency with varying actuation waveforms. Figure 1(c) shows that three modes are also required for convergent and robust results. This finding is consistent with that obtained for the static results. We also obtained the frequency-response curves in the vicinity of the second natural frequency under four excitation levels. We note that the response is slightly softening for the second mode. The fully nonlinear model accounts correctly for the inertia and bending curvature nonlinearities as well as the electrostatic nonlinearity.

References

[1] W Lacarbonara (2013) Nonlinear structural mechanics: theory, dynamical phenomena and modeling. Springer, New York.

[2] M Younis, E Abdel-Rahman, and A Nayfeh. (2003) A reduced-order model for electrically actuated microbeam-based MEMS. *J. Microelectromechanical Syst* **12**.5 : 672-680.