Explosive death transitions in a complex network of chaotic systems.

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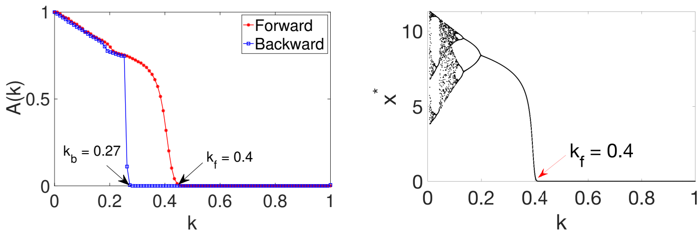
Abstract. Explosive death transitions in a mean-field coupled network of Rossler system is investigated. The network behaviour is characterised using amplitude order parameter. It is observed that the amplitude order parameter varies smoothly in the forward direction whereas, in the backward direction, it makes an explosive transition. The forward transition point has been derived analytically which matches with the numerical simulations.

Introduction

The route to synchronisation can be abrupt which is termed as explosive synchronization (ES) [1] - are defined as first order transitions and are characterised by hysteretic behaviour. Amplitude death (AD) is a type of oscillation suppression in which the oscillators stabilise to a steady state which is also the steady state of the uncoupled constituents of the network. ES where the network of coupled oscillators transition from the oscillatory state to AD state is termed as explosive death (ED). The focus of this study is on investigating analytically the mechanisms which lead to ED transitions in a mean field coupled complex network of Rossler system. The derivations make no assumption about the topology or the form or order of the oscillator units, apart from that they are assumed to undergo chaotic oscillations. Consider a network of identical chaotic systems, where each node comprises of a Rossler system. The equations of motion for the j-th node, coupled via a mean-field diffusion, is given by

$$\dot{x}_j = -y_j - z_j + \frac{kQ}{d_j}\left[\sum_{s=1}^{N} g_{js}x_s\right] - kx_j, \dot{y}_j = x_j + ay_j, \dot{z}_j = b + z_j(x_j - c). \tag{1}$$

Here, x_j, y_j, z_j are the three state variables of the j-th node, a, b, c are the system parameters, k is the strength of the coupling, Q is the density of the mean field, g_{js} is a function that represents the coupling weight between the j-th and s-th system in the network and d_j is the degree of node j. The amplitude order parameter [2] is given by $A(k) = \frac{a(k)}{a(0)}$,where, $a(k) = \frac{1}{N}\sum_{i=1}^{N}\left((x_{i,\max})_t - (x_{i,\min})_t\right)$ and $a(0)$ is the amplitude when all the nodes are uncoupled ($k = 0$).



(a) **(b)**

Figure 1: (a) Variation of $A(k)$ with k for Rossler system; $a = b = 0.2$, $c = 5.7$, $Q = 0.5$ and $N = 100$. (b)Bifurcation diagram for Rossler attractor; $a = b = 0.2$, $c = 5.7$, $Q = 0.5$ and $N = 100$

Results and Discussion

From Fig. 1(a), it is seen that in the forward direction, the variation of $A(k)$ with k is smooth and the AD state is attained at $k = 0.4$. As k is decreased adiabatically, $A(k)$ undergoes an explosive transition from the AD state. Since the transition is a second order in the forward direction and first order in the backward direction, it can be said that $A(k)$ exhibits a semi-explosive death behaviour. Fig.1(b) shows the bifurcation diagram for the j-th Rossler attractor with k (varied in the forward direction) as the bifurcation parameter. As k is increased, the system enters into chaotic state. When k is further increased, the system transitions to a period-4 oscillations and subsequently to period-2 oscillations. For $k > 0.2$, the amplitude gradually reduces and reaches the AD state for $k \geq 0.4$.

References

[1] Zhang, X., Hu, X., Kurths, J., Liu, Z. (2013) Explosive synchronization in a general complex network. *Phys. Rev. E* **88(1)**: 010802.

[2] Verma, U. K., Sharma, A., Kamal, N. K., Kurths, J., Shrimali, M. D. (2017) Explosive death induced by mean–field diffusion in identical oscillators. *Scientific Reports* **7(1)** 1-7.