Global dynamical analysis of age-structured population model

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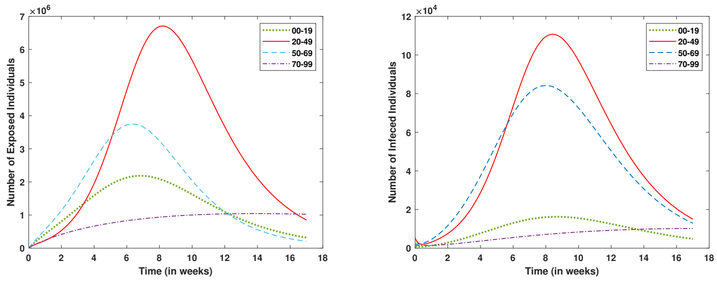
Abstract. The dynamics of the transmission and spread of infectious diseases are eminently intricate, mainly due to the heterogeneity of the host population. The patterns of interactions among various age groups can be different, which generates a significant degree of heterogeneity. Further, the treatment rate of an infectious disease plays a vital role in decreasing the spread due to the limited treatment facilities. Therefore, it is essential to involve age distribution and saturated treatment rates to model the future disease burden. This paper investigates an age-structured epidemic model incorporating a saturated treatment function. The expression for the basic reproduction number and conditions for the global stability of the system are derived via the graph-theoretic approach. It is observed that the disease-free equilibrium is globally stable if $R_0 \leq 1$ while an endemic equilibrium exists uniquely if $R_0 > 1$. The numerical simulation is demonstrated to illustrate the results.

Results and discussion

In this study, the global dynamics of an SEIR class of epidemic models with discrete age-structure and saturated treatment function is investigated. The expression for the basic reproduction number R_0 is derives using a graph-theoretic form of Gaussian elimination method. It is stated that the basic reproduction number plays the role of a sharp threshold for both the local and global asymptotic stability of each equilibria. More specifically, the disease free equilibrium M_0 is globally asymptotically stable if $R_0 \leq 1$, and if $R_0 > 1$ and the digraph associated to the disease transmission matrix is strogly connected, then the unique endemic equilibrium M^* is globally asymptotically stable under mild condition, which is the main result of this work.

Further, to facilitate the understanding of obtained theoretical results, several numerical simulations are performed vividly. We applied our age-stratified epidemic model incorporating limited treatment facilities to study the response of republic of Italy to the second wave of COVID-19 outbreak. For this purpose, the total population is divided into the following age-groups: $(00-19)years$, $(20-49)years$, $(50-69)years$, $(70-99)years$. The daily new reported cases for the republic of Italy were extracted from WHO situation reports [1] for the period 07th September 2020 to 27th December 2020. The proposed model is fitted to the extracted data and the unknown parameters are estimated. The contact patterns across different age groups have a great degree of heterogeneity. During COVID-19, these mixing patterns are known to be crucial determinants for the model outcome and highly assortative with age. In most mathematical models, the number of people a person contacts per day is assumed to be a constant or follow a particular pattern. In this work, these age-dependent contact rates are estimated via a paper-diary methodology [2], based on a population-prospective survey in European countries. For this purpose, relevant contact data from the POLYMOD (Improving Public Health Policy in Europe through Modelling and Economic Evaluation of Interventions for the Control of Infectious Diseases) study is used. The time profile shows in figure 1 clearly depicts the role of different age-groups in the spread of disease. For the infected individuals lying in the age-group (20-49) years and (50-69) years, the susceptibility to infection is approximately double compared to the exposed population is an important observation of this work. The rapidity of the evolved COVID-19 outbreak in 2020 makes us realize the subverted situation of our world in times of emergency. The results demonstrated in this study helps the policy makers in analyzing the disease severity among different age-groups and therefore in adapting the effective control measures.



(a) (b)

Figure 1: Susceptibility to infection for Exposed and Infected classes varies by age.

References

[1] World Health Organisation. **https://covid19.who.int/region/euro/country/it**.

[2] M.I. Meltzer, M. Gambhir, et al. (2015) Standarizing scenarios to assess the need to respond to an influenza pandemic *Clin, Infect. Dis.* **60**:S1-S8.