A Bayesian compressive sampling technique for determining the equations of motion of nonlinear structural systems

G. D. Pasparakis^{*}, V. C. Fragkoulis^{*}, I. A. Kougioumtzoglou^{**} and M. Beer^{*} ^{*}Institute for Risk and Reliability, Leibniz University Hannover, Hannover, Germany ^{**}Department of Civil Engineering and Engineering Mechanics, Columbia University, New York, USA

Abstract. A technique based on Bayesian compressive sampling is developed for determining the governing equations of stochastically excited structural systems exhibiting diverse nonlinear behaviors and/or following a fractional derivative modeling. This is done by relying on measured data and by employing sparsity-promoting optimization algorithms for determining the active coefficients in an expansion basis approximating the system dynamics. A significant advantage of the technique relates to the fact that the uncertainty associated with the model estimate is also quantified.

Introduction

A novel paradigm of data-driven model discovery has emerged in recent years. In this context, it can be argued that the identified model should exhibit sparsity in the sense that the fewest possible terms are considered for the description of the system dynamics. The rationale relates to the fact that the dynamics of most physical systems can be described accurately by considering only very few relevant terms in an appropriate expansion basis; thus, rendering the governing equations sparse in a high-dimensional nonlinear function space. In this regard, several approaches for sparse identification of nonlinear dynamics based on compressive sampling concepts and tools have been proposed recently [1].

Results and discussion

In this paper, a technique based on Bayesian compressive sampling [2] is developed for determining the governing equations of stochastically excited structural systems exhibiting diverse nonlinear behaviors and/or following a fractional derivative modeling. This is done by relying on measured data and by utilizing a state-variable formulation of the system governing equations. Further, considering an expansion basis for approximating the nonlinear system dynamics leads to a non-square system of equations. This is solved based on sparsity-promoting optimization algorithms for determining the active coefficients in the expansion basis. Compared to alternative state-of-the-art schemes that yield deterministic estimates for the expansion coefficient vector, the herein developed technique is capable also of quantifying the uncertainty associated with the model estimate; thus, providing a measurable confidence degree when employing the technique as a predictive tool; see also Fig.1. The reliability of the technique, even in cases of highly limited/incomplete measured data, is demonstrated by considering various numerical examples.



Figure 1: Schematic representation of the Bayesian Compressive Sampling technique for determining governing equations of nonlinear dynamical systems

References

[1] Brunton S., Kutz J. N. Kutz (2022) Data-driven science and engineering: Machine learning, dynamical systems, and control. Cambridge University Press.

[2] Kougioumtzoglou I. A., Petromichelakis I., Psaros A. F. (2020). Sparse representations and compressive sampling approaches in engineering mechanics: A review of theoretical concepts and diverse applications, *Prob. Eng. Mech.* **61**: 103082.