Resonance bifurcations in a discrete-time predator-prey system

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Abstract. This paper studies bifurcation analysis and resonances in a discrete-time model analytically and numerically. The local stability conditions of all the fixed points in the system are determined. Here, codim-1 and codim-2 bifurcation, including multiple and generic bifurcations in the discrete model, are explored. The model undergoes fold bifurcation, flip bifurcation, Neimark-Sacker bifurcation and resonances bifurcation of codimension two at different fixed points. Using critical normal form theorem and bifurcation theory, this study obtains the normal form coefficients to confirm the nondegeneracy of codim-1 and codim-2 bifurcations in the model. The numerical simulation gives a wide range of periodic cycles and bifurcation in the system. In the system, NSB signifies that both species can fluctuate near critical parameter values and stable fluctuations seem. The resonance bifurcation in the discrete-time map indicates that both species coincide till order 4 in stable periodic cycles near some critical parametric values.

Introduction

In ecology, the interaction between distinct species causes conflict, cooperation and consumption. The prey-predator system is the most fundamental linkage among them. Almost a century ago, the predator and prey populations had many variations based on their experimental evidence. The Lotka-Volterra model also serves as a general framework for describing other types of nonlinear ecosystem interactions such as competition, scavenging, and mutualism. The study of their dynamical behaviors has drawn the attention of many mathematical biologists [1, 3, 2].

Results and discussion

In this work, we obtained our discretized model by deploying the piecewise constant argument approach in the model, discussed. we explored local stability for all fixed points. The system is explored for different codim-1 and codim-2 bifurcations by using critical normal form coefficient method. The codim-2 bifurcations such as resonance 1:2(R2), resonance 1:3(R3) and resonance 1:4(R4) are occurred under some non-degenerate conditions. An extensive numerical simulation is presented to substantiate the analytical findings.
Moreover, the 1:4 resonance bifurcation demonstrates that both species coexist till order 4 in stable periodic cycles near some critical parametric values. Ecologically, the prey-predator species coexist up to the fourth order in the stable high periodic cycle.

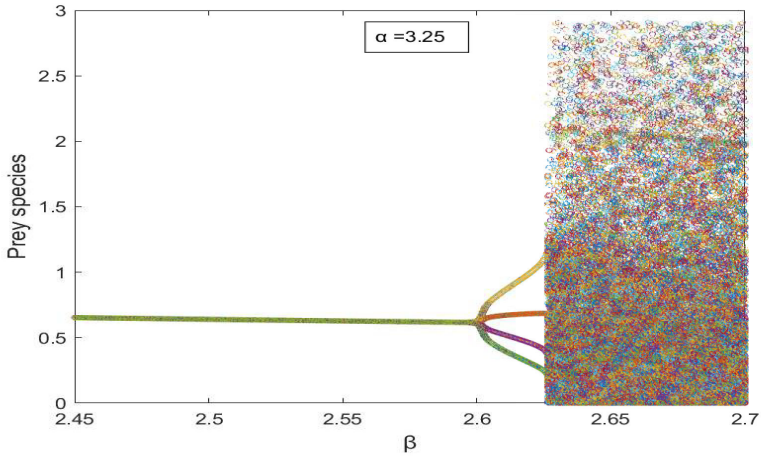

Figure 1: 1;4 (R4) Resonance bifurcation diagram with respect to β when $\alpha = 3.25$

References

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