Frequency-Energy Analysis of Coupled Linear Oscillator with Unsymmetrical Nonlinear Energy Sink

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Abstract. The underlying nonlinear dynamical behaviour of linear oscillator (LO) attached with a nonlinear energy sink (NES), in which unsymmetrical coupling force is employed, is investigated here on the FEP. The nonlinear force incorporates purely cubic stiffness element on one side of the equilibrium position and linear restoring coupling stiffness on the other side. The obtained FEP of this system reveals different kinds of backbone curves. Accordingly, the FEP shows five new continuous backbones of 1:1 resonance between the LO and the UNES at frequency levels below the LO natural frequency.

Introduction

The frequency energy plot (FEP) analysis has significant impact on revealing the underlying nonlinear dynamical behaviour of LO-NES oscillations. The periodic motion of stiffness-based structure-NES systems has been extensively studied in the literature on the harmonic nonlinear normal modes (NNMs) backbone curves and their associated subharmonic branches [1-5]. The FEPs accompanied with the superimposed wavelet frequency spectrum content have been also employed to confirm the existence of different kinds of resonance captures between the LO and the NES oscillations. The resonance captures on the FEP backbones and their associated subharmonic branches have verified the rapid and passive targeted energy transfer (TET) from the primary system into the NES attachment [1-3]. Accordingly, the FEP analysis is generated here for coupled LO with unsymmetrical nonlinear energy sink (UNES).

The LO-UNES System and the FEP

The considered LO in Figure 1a is coupled with UNES of cubic stiffness on one side and linear stiffness on the other side. The FEP of this LO-UNES system has been obtained at $M = 1$ kg, $m = 0.05$ kg $k_1 = 1$ N/m, $k_{res} = 0.03$ N/m, $k_{nl} = 1$ N/m^3 and zero damping content using the continuation method in [4,5]. The free-response of the Hamiltonian equations of motion of the LO-UNES system is obtained at the given physical parameters for zero velocities and nonzero displacements via numerical simulation. Compared with the NES with purely cubic stiffness force in the literature, the FEP in Figure 1b of the LO-UNES system shows five new continuous backbones named as *a*, *b*, *c*, *d* and *e* at frequency levels below the LO natural frequency. These backbones overlap with the subharmonic branches that appear due to the cubic stiffness effect in the UNES. At these backbones, it is observed that the oscillation is dominated by the linear effect of the UNES below some energy threshold whereas it becomes dominated by the nonlinear effect above that threshold.

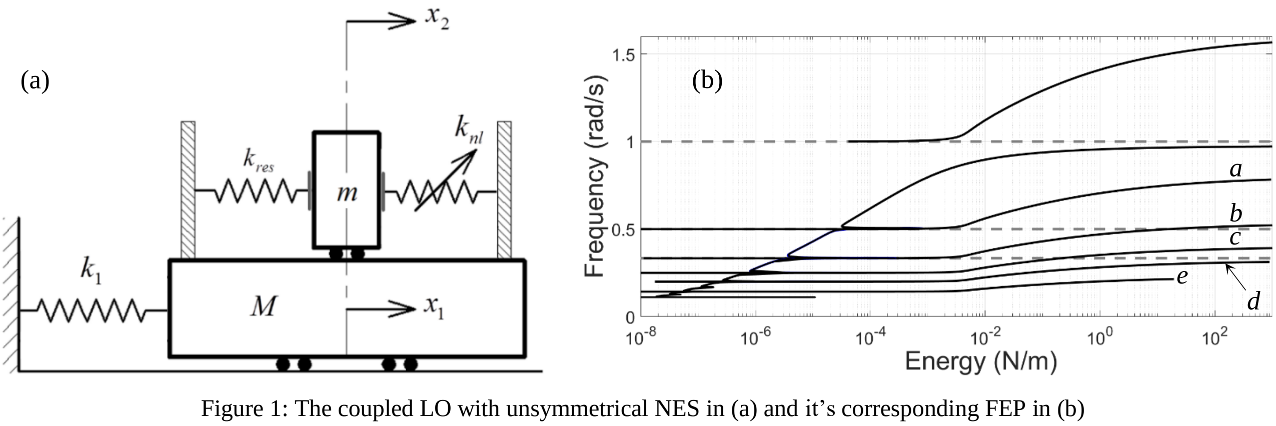

Figure 1: The coupled LO with unsymmetrical NES in (a) and it's corresponding FEP in (b)

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