On the velocity-based description in dynamic analysis of three-dimensional beams

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Abstract. In numerical formulations of three-dimensional beams the choice of primary interpolated variables is highly important for the efficiency and accuracy of the method. The crucial idea exploited in our approach is to employ velocity and angular velocity vectors in their suitable component descriptions to set the discrete computational model.

Introduction

Many computational challenges in dynamic modelling of frame-like structures are directly related to the properties of configuration space of three-dimensional beams which typically incorporates three-dimensional rotations. Since rotations form a non-commutative multiplicative group their computational treatment requires a special care. In contrast, the measures for their rate of change – the angular velocities are additive quantities when expressed with respect to the moving basis. The crucial idea is thus to employ velocities in fixed frame description and angular velocities in moving frame description as the primary unknowns of numerical model.

Kinematic compatibility

Another important property of continuous system, exploited in our work, is the direct relation between the strains and the velocities, called the *compatibility equations* [1]. In Cosserat rod theory the resultant strain measures at the centroid of each cross-section are directly introduced and expressed with kinematic variables by the first order differential equations

$$\boldsymbol{\Gamma} = \widehat{\mathbf{q}}^* \circ \mathbf{r}' \circ \widehat{\mathbf{q}} + \boldsymbol{\Gamma}_0, \qquad \mathbf{K} = 2\widehat{\mathbf{q}}^* \circ \widehat{\mathbf{q}}', \tag{1}$$

where $\boldsymbol{\Gamma}$ and $\mathbf{K}$ denote the translational and rotational strain, respectively, both expressed with respect to the local basis, while position vector $\mathbf{r}$ and rotational quaternion $\widehat{\mathbf{q}}$ are expressed in the fixed basis. It is important to observe that strains, velocities, and angular velocities are mutually dependent. Their direct relation is obtained by comparing mixed partial derivatives, which gives

$$\dot{\boldsymbol{\Gamma}} = \widehat{\mathbf{q}}^* \circ \mathbf{v}' \circ \widehat{\mathbf{q}} + (\boldsymbol{\Gamma} - \boldsymbol{\Gamma}_0) \times \boldsymbol{\Omega}, \qquad \dot{\mathbf{K}} = \boldsymbol{\Omega}' + \mathbf{K} \times \boldsymbol{\Omega} \tag{2}$$

with $\mathbf{v}$ denoting velocity and $\boldsymbol{\Omega}$ the angular velocity. In our approach we satisfy the kinematic compatibility equations (2) with the same accuracy as the governing equations. The discrete kinematic compatibility equations can be completely harmonized with the energy conservation demands, which results in robustness and long-term stability of the overall algorithm.

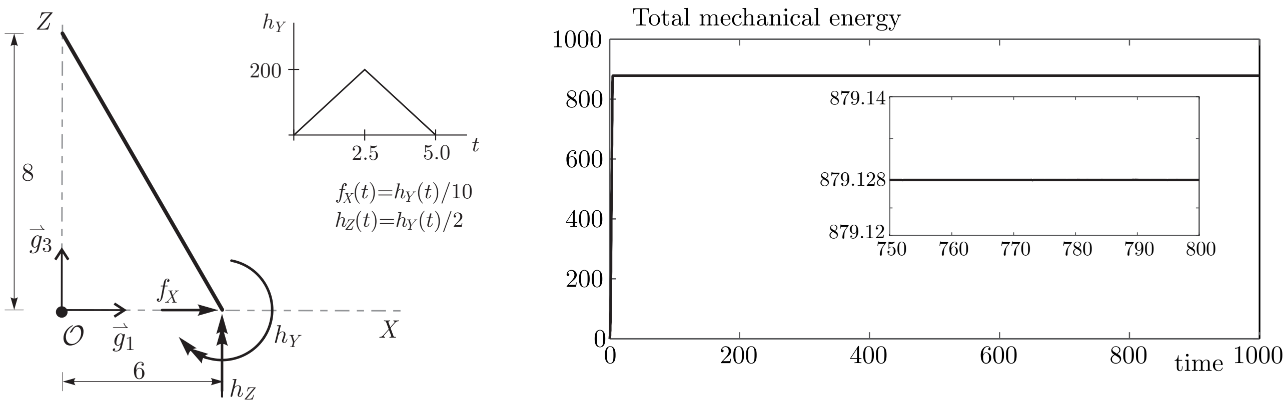

Figure 1: Free flight of a flexible beam: problem description (left) and total mechanical energy (right).

Figure 1 shows the performance of the present approach for the benchmark problem of flying flexible beam presented by Simo and Vu-Quoc [2] and characterized by very large displacements and rotations. The long-term stability and the energy preservation using our model can be observed.

References

[1] Antman S. S. (2003) Invariant dissipative mechanisms for the spatial motion of rods suggested by artificial viscosity. *J. Elast.*, **70**(1-3):55–64.

[2] Simo, J.C. , Vu-Quoc, L. (1988) On the dynamics in space of rods undergoing large motions - a geometrically exact approach, *Comput. Meth. Appl. Mech. Eng.* **66**(2), 125.