Time-Periodic perturbation leading to chaos in a planar memristor oscillator having a Bogdanov-Takens bifurcation

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Abstract. We consider a memristive circuit consisting of a locally-active current-controlled memristor, a compensation inductor and a bias resistor, which is modeled by a three-parameter two-dimensional system of ordinary differential equations. The system presents periodic oscillations, which arise at a Hopf bifurcation. We show that these oscillations evolve into a homoclinic orbit, in a Bogdanov-Takens bifurcation type scenario. By adding a small time-periodic excitation to the circuit, we obtain complex dynamical behavior, such as quasi-periodic and chaotic oscillations.

Introduction

The *memristor*, a nonlinear resistor with memory, is considered the fourth fundamental circuit element, besides resistor, capacitor and inductor. It was theoretically proposed in 1971 by Leon Chua [1] and its physical realization was possible only in 2007 [2]. Since then, the memristor attracts much interest from the academia and industry, due to its potential applications in several technological areas, like the construction of nonvolatile memories, logic operation circuits, artificial neural networks and chaotic oscillations [3]. Complex behavior arising in memristive systems can be generated by the locally-active characteristics of memristors [4]. At present, there are some researches on locally-active memristors and its interaction with other fundamental circuit elements [4, 5], but there is a lack of research on the mechanism creation of periodic and chaotic oscillations caused by them. We propose a new mechanism to obtain such a complex behaviors in memristive systems.

Results and discussion

We consider the periodic oscillator memristive circuit proposed in [5], consisting of three elements: a locally-active current-controlled memristor, a compensation inductor and a bias resistor. The circuit is modeled by the following system of ordinary differential equations

$$\dot{x} = 250[2x - x^2 - 2x^3 + x^4 + (5.4 - 2.8x)y], \qquad \dot{y} = \frac{1}{L}\left[(S-y)R + 1.5(x-0.5)y\right]. \tag{1}$$

where L, S and R are control parameters, x and y are state variables, which are proportional to the internal state of memristor and to the current in the circuit, respectively. The dynamics of system (1) was studied in [5], where the authors shown the occurrence of periodic oscillations, arising at a Hopf bifurcation, when the parameter L is varied. Then, they added a capacitor to the circuit, generating another state equation in system (1), in order to obtain a chaotic three-dimensional system [5]. In this work, we show that the periodic oscillations of system (1) tend to a homoclinic orbit, showing a Bogdanov-Takens bifurcation type scenario (see Fig. 1).

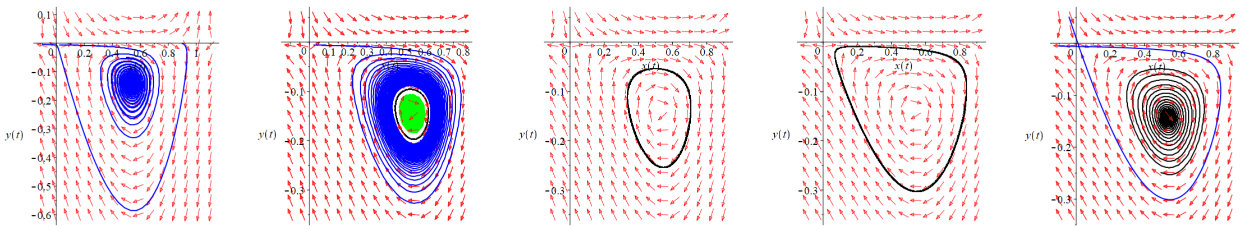

Figure 1: Bogdanov-Takens bifurcation scenario of the solutions of system (1) obtained varying the parameter L.

In order to obtain complex behavior, such as quasi-periodic, canards and chaotic oscillations, we add into system (1) a small external time-periodic excitation of the form $f(A, \omega, t) = A\cos(\omega t)$, obtaining a non-autonomous time-periodic system. With this procedure, we show that chaotic dynamics can be obtained in a memristive circuit with a locally-active memristor through the input of a periodic stimulus, instead of adding a new element to the circuit, which grows the dimension of the related differential system, as is often made in literature [5]. As far as we know, and also after a Google search, it is the first time that Bogdanov-Takens bifurcation and its time-periodic perturbation are considered in the study of memristive circuits and systems as a mechanism to generate complex dynamical behavior.

References

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