Multiple Equilibrium States in Large Array Resonators

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Abstract. This work considers the response of a globally coupled array of oscillators, each with cubic nonlinear stiffness, in the presence of global reactive and dissipative coupling. Based on the continuum formulation for this system first presented in C. Borra et al (Journal of Sound and Vibration, 393:232–239, 2017), the individual resonators are excited to sufficient amplitude to allow for multiple coexisting equilibrium distributions. The method of multiple scales is then applied to the system to describe evolution equations for the amplitude and phase of each resonator, and the equilibrium structure of the system is studied as the reactive and dissipative coupling parameters are varied. For specific families of the equilibrium distributions two-parameter bifurcation sheets can be constructed., and these sheets are connected as individual resonators transition between different branches for the corresponding individual resonators.

Introduction

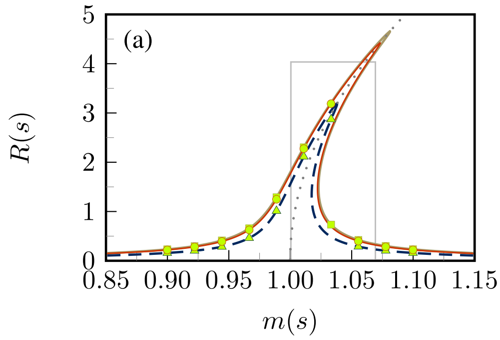
The equations of motion for a discrete system of coupled resonators [1] can be written as

$$m_i\,\ddot{z}_i + c_i\,\dot{z}_i + k_{1,i}\,z_i + k_{3,i}\,z_i^3 - \frac{\tilde{\alpha}}{N}\sum_{j=1}^{N}\dot{z}_j - \frac{\tilde{\beta}}{N}\sum_{j=1}^{N} z_j = \tilde{f}(t), \tag{1}$$

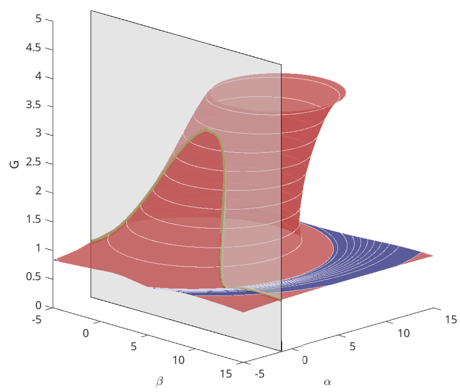
where each individual resonator in the N-element array is characterized by the index i and has an associated displacement z_i. The parameters $\tilde{\alpha}$ and $\tilde{\beta}$ characterize the magnitude of the dissipative and reactive global coupling respectively, and finally each resonator is subject to an external time-dependent excitation represented by $\tilde{f}(t)$. This system is analyzed using the method of multiple scales applied in a continuum limit of the population, first introduced in Borra et al. [2].

Results and Discussion

As illustrated in Figure 1a, for given values of the coupling parameters $(\tilde{\alpha}, \tilde{\beta})$, coexisting stationary distributions of the population of resonators exist. In the figure three distinct equilibrium populations are shown for $N = 10$ resonators, each with given mass detuning. Further, distinct two-parameter bifurcation sets can be obtained for specific population characteristics as the coupling varies, an example of which is shown in Figure 1b. The resulting families of bifurcation sets can be pieced together to then understand the overall bifurcation structure of the system.



(a) Coexisting equilibrium distributions, $N = 10$; each distribution is indicated by a different line and marker style (most notable at $i = 7$).



(b) Bifurcation Sheet

Figure 1: Equilibrium Structure

References

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[2] Borra C., Pyles C.S., Wetherton B.A., Quinn D.D., Rhoads J.R. (2017) The dynamics of large-scale arrays of coupled resonators. *Journal of Sound and Vibration*, 392:232–239, 2017.

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